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FOR SPACE STANDARDIZATION

Space engineering

Thermal design handbook - Part 1: View factors

ECSS Secretariat
ESA-ESTEC
Requirements & Standards Division
Noordwijk, The Netherlands

Foreword

This Handbook is one document of the series of ECSS Documents intended to be used as supporting material for ECSS Standards in space projects and applications. ECSS is a cooperative effort of the European Space Agency, national space agencies and European industry associations for the purpose of developing and maintaining common standards.

The material in this Handbook is a collection of data gathered from many projects and technical journals which provides the reader with description and recommendation on subjects to be considered when performing the work of Thermal design.

The material for the subjects has been collated from research spanning many years, therefore a subject may have been revisited or updated by science and industry.

The material is provided as good background on the subjects of thermal design, the reader is recommended to research whether a subject has been updated further, since the publication of the material contained herein.

This handbook has been prepared by ESA TEC-MT/QR division, reviewed by the ECSS Executive Secretariat and approved by the ECSS Technical Authority.

Disclaimer

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1 Scope

In this Part 1 of the spacecraft thermal control and design data handbooks, view factors of diffuse and specular thermal surfaces are discussed.

For diffuse surfaces, calculations are given for radiation emission and absorption between different configurations of planar, cylindrical, conical, spherical and ellipsoidal surfaces for finite and infinite surfaces.

For specular surfaces the affect of reflectance on calculations for view factors is included in the calculations. View factors for specular and diffuse surfaces are also included.

The Thermal design handbook is published in 16 Parts

ECSS-E-HB-31-01 Part 1	Thermal design handbook – Part 1: View factors
ECSS-E-HB-31-01 Part 2	Thermal design handbook – Part 2: Holes, Grooves and Cavities
ECSS-E-HB-31-01 Part 3	Thermal design handbook – Part 3: Spacecraft Surface Temperature
ECSS-E-HB-31-01 Part 4	Thermal design handbook – Part 4: Conductive Heat Transfer
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ECSS-E-HB-31-01 Part 15	Thermal design handbook – Part 15: Existing Satellites
ECSS-E-HB-31-01 Part 16	Thermal design handbook – Part 16: Thermal Protection System

2 References

ECSS-S-ST-00-01	ECSS System - Glossary of terms
ECSS-E-HB-31-01 Part 3	Thermal design handbook – Part 3: Spacecraft Surface Temperature

All other references made to publications in this Part are listed, alphabetically, in the **Bibliography**.

3 Terms, definitions and symbols

3.1 Terms and definitions

For the purpose of this Standard, the terms and definitions given in ECSS-S-ST-00-01 apply.

3.2 Symbols

A_i	surface area of the i-th surface, [m^2]
B_i	energy flux leaving surface, *i*. often called radiosity, [W.m^{-2}]
F_{ij}	view factor from diffuse surface, A_i to diffuse surface, A_j
$F_{(i1,i2,\ldots in)(j1,j2,\ldots jn)}$	view factor from the ensemble of diffuse surfaces, A_{i1}, A_{i2},...A_{in} to the ensemble of diffuse surfaces, A_{j1}, A_{j2},...A_{jn}
F_{ij}^s	view factor from specular surface A_i to specular surface A_j
H_i	energy flux incident on surface *i*, [W.m^{-2}]
K_{i2}	term which appears in the expression for the view factor between elements of parallel plates, $K_{i2} = A_i F_{ii'}$
$K_{mn}(i,j,k,p,q,\ldots)$	fraction of the radiative energy leaving A_m which reaches A_n after *i* perfectly specular reflections from surface A_i, *j* from surface A_j, *k* from surface A_k,...
S	distance between two differential elements, [m]
T	temperature, [K]
β_i	angle from normal to surface *i*, [angular degrees]
ε	hemispherical emittance of a (diffuse-gray) surface
ρ^d	hemispherical diffuse reflectance of a (diffuse-gray) surface

ρ^s	specular reflectance of a (gray) surface, it is assumed to be independent of incident angle
σ	Stefan-Boltzmann constant, S = 5,6697x10^{-8} W.m^{-2}.K^{-4}

Other symbols, mainly used to define the geometry of the configurations, are introduced when required.

4 Diffuse surfaces

4.1 General

The view factor, F_{12}, between the diffuse surface A_1 and A_2, is the fraction of the energy leaving the isothermal surface A_1 that arrives at A_2.

If the receiver surface is infinitesimal, the view factor is infinitesimal for both infinitesimal and finite emitting surfaces, and is given by the expression

$$dF_{12} = \frac{\cos\beta_1 \cos\beta_2}{\pi S^2} dA_2 \qquad [4\text{-}1]$$

when both surfaces are infinitesimal, and by

$$dF_{12} = \frac{dA_2}{A_1} \int_{A_1} \frac{\cos\beta_1 \cos\beta_2}{\pi S^2} dA_1 \qquad [4\text{-}2]$$

when A_1 is finite.

If the receiver surface is finite, the view factor is finite for both infinitesimal and finite emitting surfaces, and is given by the expression

$$F_{12} = \int_{A_2} \frac{\cos\beta_1 \cos\beta_2}{\pi S^2} dA_2 \qquad [4\text{-}3]$$

when A_1 is infinitesimal, and by

$$F_{12} = \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos\beta_1 \cos\beta_2}{\pi S^2} dA_2 dA_1 \qquad [4\text{-}4]$$

when A_1 is finite.

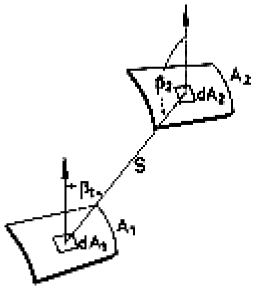


Figure 4-1: Geometric notation for view factors between diffuse surface.

Regardless of which surfaces are considered, their view factors satisfy the following reciprocity relation:

$A_1F_{12} = A_2F_{21}$

If we consider the diffuse surfaces A_1, A_2 and A_3, the view factor between the surfaces A_1 and $A_2 + A_3$ is

$F_{1(2,3)} = F_{12} + F_{13}$,

when the receiver surface is formed by two surfaces, and

$$F_{(2,3)1} = \frac{A_2F_{21} + A_3F_{31}}{A_2 + A_3} \quad [4\text{-}5]$$

when the emitting surface is formed by two surfaces. notice that the notation $F_{1(2,3)}$ and $F_{(2,3)1}$ will be used in the following data sheets.

When an enclosure of N surfaces $A_1, A_2, \ldots, A_n$ is considered, their view factors satisfy the relation

$$\sum_{j=1}^{N} F_{ij} = 1 \quad [4\text{-}6]$$

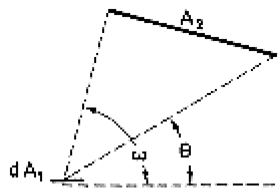
for any surface A_i. This relationship results from the fact that the overall heat transfer in the enclosure should be zero.

4.2 Infinitesimal to finite surfaces

4.2.1 Planar to planar

4.2.1.1 Two-dimensional configurations

A plane point source dA_1 and any surface A_2 generated by an infinitely long line moving parallel to itself and to the plane dA_1.



Formula:

$$F_{12} = \frac{1}{2}(\cos\theta - \cos\omega) \quad [4\text{-}7]$$

References: Hamilton & Morgan (1952) [15], Moon (1961) [26], Kreith (1962) [22]

Comments:

Notice that F_{12} is independent of the shape of A_2 for given values of θ and ω.

View factors for several configurations may be obtained as a particular case of this one. An example is shown in the next page.

A plane point source dA_1 and any infinite plane A_2 with the planes of dA_1 and A_2 intersecting at an angle θ.

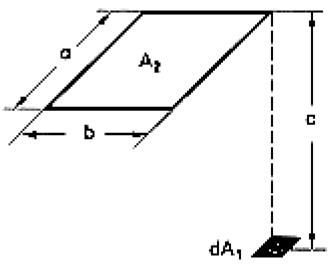
Formula:

$$F_{12} = \frac{1}{2}(1 + \cos\theta) \quad [4\text{-}8]$$

References: Hamilton & Morgan (1952) [15], Moon (1961) [26], Kreith (1962) [22]

4.2.1.2 Point source to rectangle

A plane source dA_1 and a plane rectangle A_2 parallel to the plane of dA_1 (see sketch). The normal to dA_1 passes through one corner of A_2.



$x = a/c$
$y = b/c$

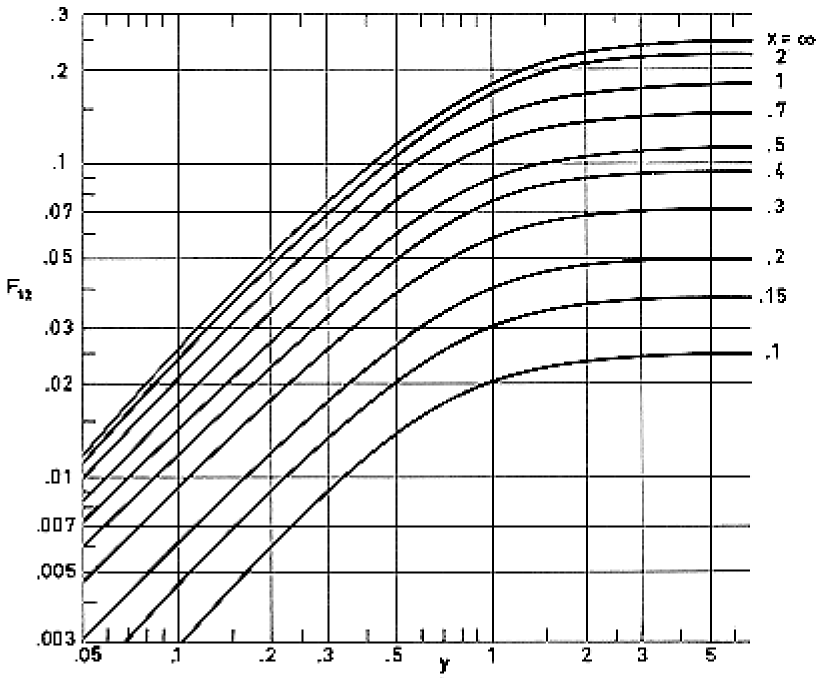
Formula:

$$F_{12} = \frac{1}{2\pi}\left(\frac{x}{\sqrt{1+x^2}} \tan^{-1} \frac{y}{\sqrt{1+x^2}} + \frac{y}{\sqrt{1+y^2}} \tan^{-1} \frac{x}{\sqrt{1+y^2}} \right) \quad [4\text{-}9]$$

$$\lim_{y\to\infty} F_{12} = \frac{x}{4\sqrt{1+x^2}} \quad [4\text{-}10]$$

$$\lim_{x\to\infty} F_{12} = \frac{y}{4\sqrt{1+y^2}} \quad [4\text{-}11]$$

References: Hamilton & Morgan (1952) [15], Hottel (1954) [17], Jakob (1957) [19], Moon (1961) [26], Kreith (1962) [22]

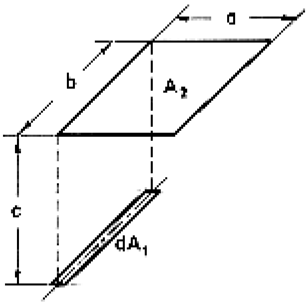


Note: non-si units are used in this figure

Figure 4-2: Values of F_{12} as a function of x and y. From Hamilton & Morgan (1952) [15].

4.2.1.3 Line source to rectangle

A line source dA_1 and a plane rectangle A_2 parallel to the plane of dA_1 with dA_1 opposite one edge of A_2.



$x = b/c$
$y = a/c$

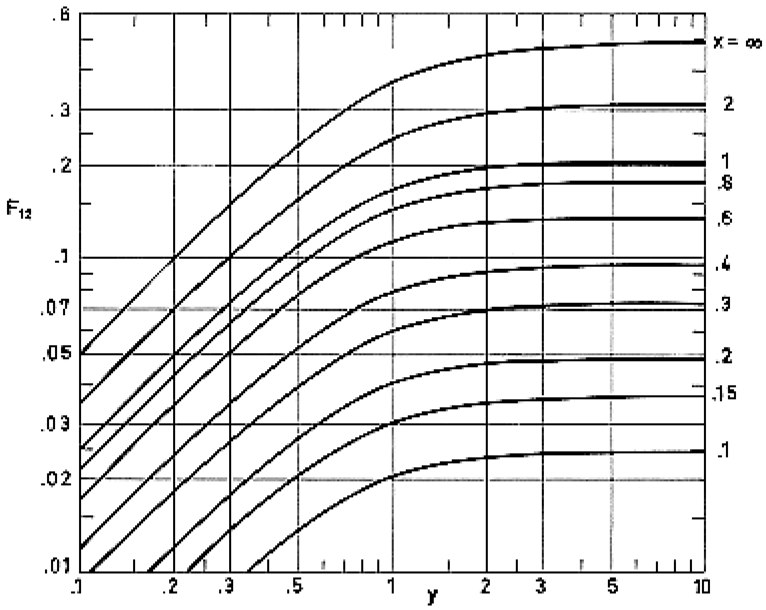
Formula:

$$F_{12} = \frac{1}{\pi x}\left(\sqrt{1+x^2}\tan^{-1}\frac{y}{\sqrt{1+x^2}} - \tan^{-1} y + \frac{xy}{\sqrt{1+y^2}}\tan^{-1}\frac{x}{\sqrt{1+y^2}} \right) \quad [4\text{-}12]$$

$$\lim_{x\to\infty} F_{12} = \frac{y}{2\sqrt{1+y^2}} \quad [4\text{-}13]$$

$$\lim_{y\to\infty} F_{12} = \frac{1}{2}\left(\sqrt{1+\frac{1}{x^2}} - \frac{1}{x} \right) \quad [4\text{-}14]$$

References: Hamilton & Morgan (1952) [15], Kreith (1962) [22]

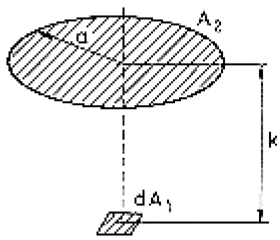


Note: non-si units are used in this figure

Figure 4-3: Values of F_{12} as a function of x and y. From Hamilton & Morgan (1952) [15]

4.2.1.4 Point source to coaxial disc or annulus

Infinitesimal surface to finite coaxial disc.



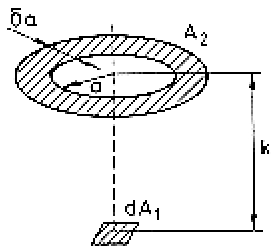
$H = a/k$

Formula

$$F_{12} = \frac{H^2}{1+H^2} \quad [4\text{-}15]$$

The following case can be obtained by differentiating the above expression.

Infinitesimal surface to very thin coaxial annulus with finite radius.



$H = a/k$
$\delta H = \delta a/k$

Formula

$$F_{12} = \frac{2H \cdot \delta H}{\left(1 + H^2\right)^2} \qquad [4\text{-}16]$$

This expression has been represented in the Figure 4-4.

Reference: Chung & Sumitra (1972) [6].

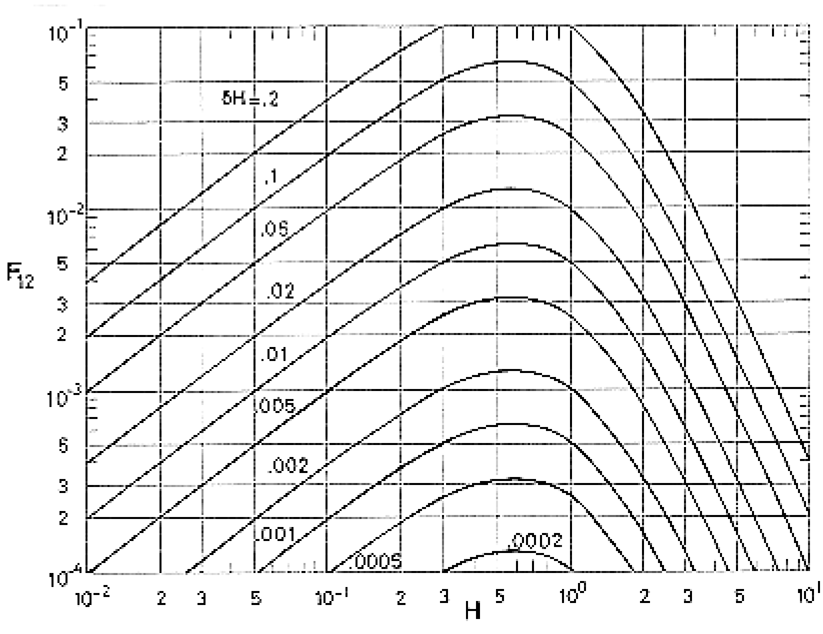
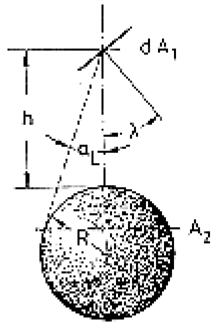


Figure 4-4: F12 vs. *H* for different values of dH. Infinitesimal surface to very thin coaxial annulus with finite radius. Calculated by the compiler.

4.2.2 Planar to spherical

One face of an elemental plate to sphere.



$H = h/R$

$\mathrm{cosec}\,\alpha_L = 1 + H$

Two cases can be distinguished:

I. The upper cap is fully visible from the plate ($\lambda + \alpha_L \leq \pi/2$).

The analytical expression for the view factor is:

$$F_{12} = \frac{\cos\lambda}{(1+H)^2} \quad [4\text{-}17]$$

II. The upper cap is partially visible from the plate ($\lambda + \alpha_L > \pi/2$).

The self-shielding of the plate destroys the ease of integration of case I. the calculations are carried out numerically, by means of the following finite polynomial:

$F_{12} = B_0 + B_1\cos\lambda + B_2\cos^2\lambda + B_3\cos^3\lambda + B_4\cos^6\lambda$.

The parameters B_i ($i = 0,\ldots,4$) are given in ECSS-E-HB-31-01 Part 3 Clause 5.1

This expression can be used in both cases although the values in the shadowed area in Figure 4-5 below have been calculated by the use of the analytical expression.

References: Cunningham (1961) [8], Kreith (1962) [22], Bannister (1965) [1], Clark & Anderson (1965) [7].

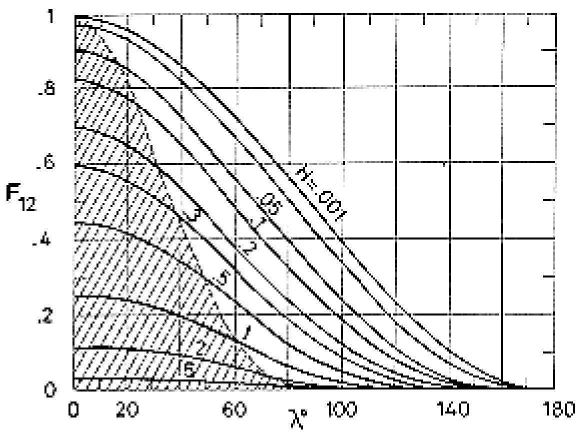
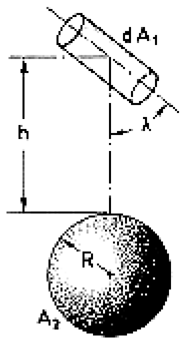


Figure 4-5: Values of F_{12} vs. λ for different values of H. The analytical expression (case I) is only valid in the shadowed region. Calculated by the compiler.

4.2.3 Cylindrical to spherical

Outer area of an infinitesimal cylindrical surface to sphere.

$H = h/R$



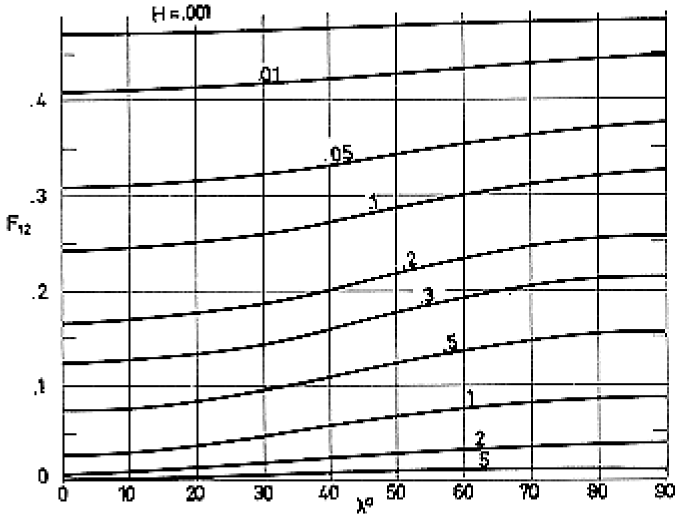
Formula:

$$F_{12} = \frac{1}{\pi^2}\int_0^{2\pi} d\phi \int_0^{\theta_o} \sqrt{1-(\cos\theta\cos\lambda+\sin\theta\sin\lambda\cos\phi)^2}\,\sin\theta d\theta \qquad [4\text{-}18]$$

where

$$\sin\theta_o = \frac{1}{1+H} \qquad [4\text{-}19]$$

References: Watts (1965) [50], Clark & Anderson (1965) [7].



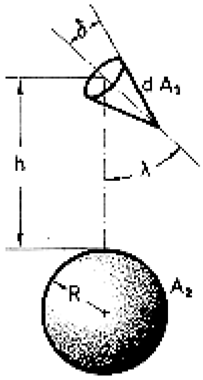
Note: non-si units are used in this figure

Figure 4-6: Values of F_{12} as a function of H and λ. Calculated by the compiler.

4.2.4 Conical to spherical

Outer area of an infinitesimal conical surface to sphere.

$H = h/R$



All results presented in the literature are obtained numerically.

References: Clark & Anderson (1965) [7].

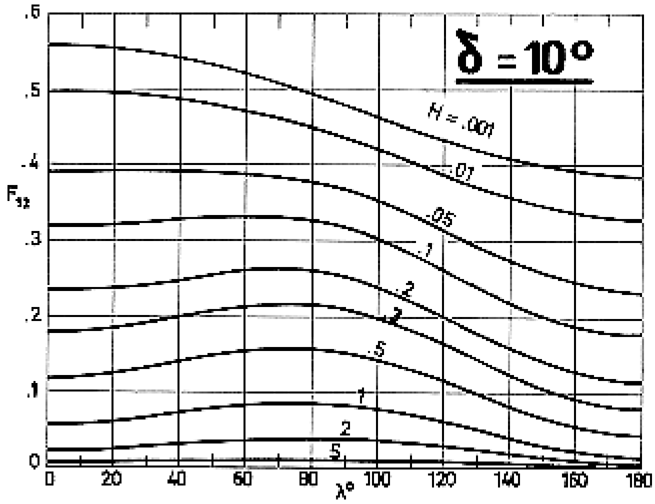


Figure 4-7: Values of F_{12} as a function of H and λ, for $\delta = 10°$. Calculated by the compiler.

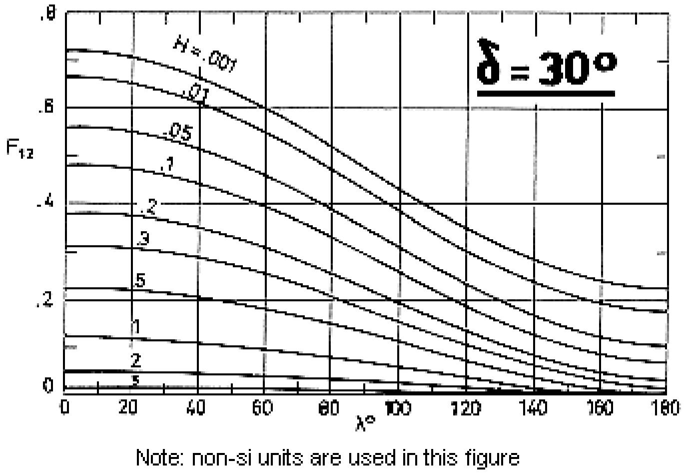


Figure 4-8: Values of F_{12} as a function of H and λ, for δ = 30°. Calculated by the compiler.

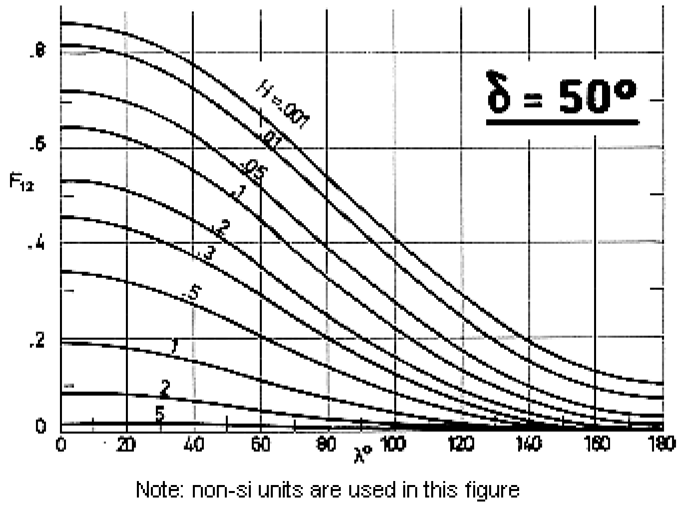
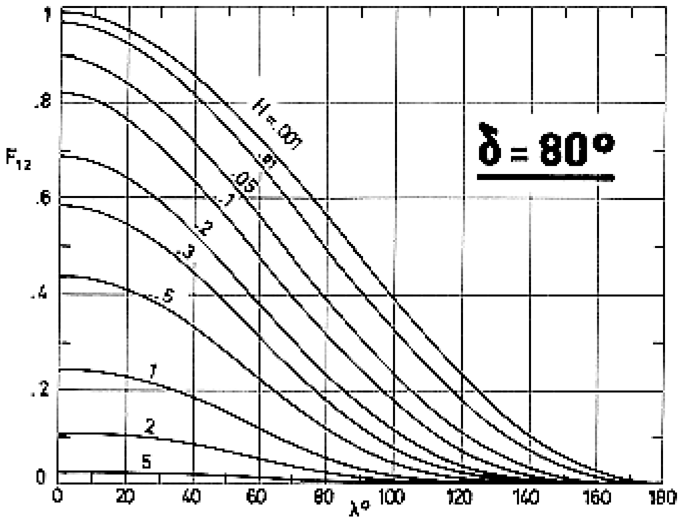


Figure 4-9: Values of F_{12} as a function of H and λ, for δ = 50°. Calculated by the compiler.



Note: non-si units are used in this figure

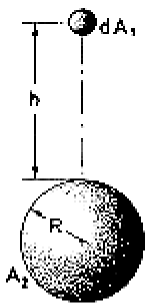
Figure 4-10: Values of F_{12} as a function of H and λ, for δ = 80°. Calculated by the compiler.

4.2.5 Spherical to spherical

4.2.5.1 Sphere to outer sphere

Infinitesimal sphere to finite sphere.

$H = h/R$



Formula:

$$F_{12} = \frac{1}{2}\left(1 - \frac{\sqrt{H^2 + 2H}}{1 + H}\right) \qquad [4\text{-}20]$$

Reference: Watts (1965) [50].

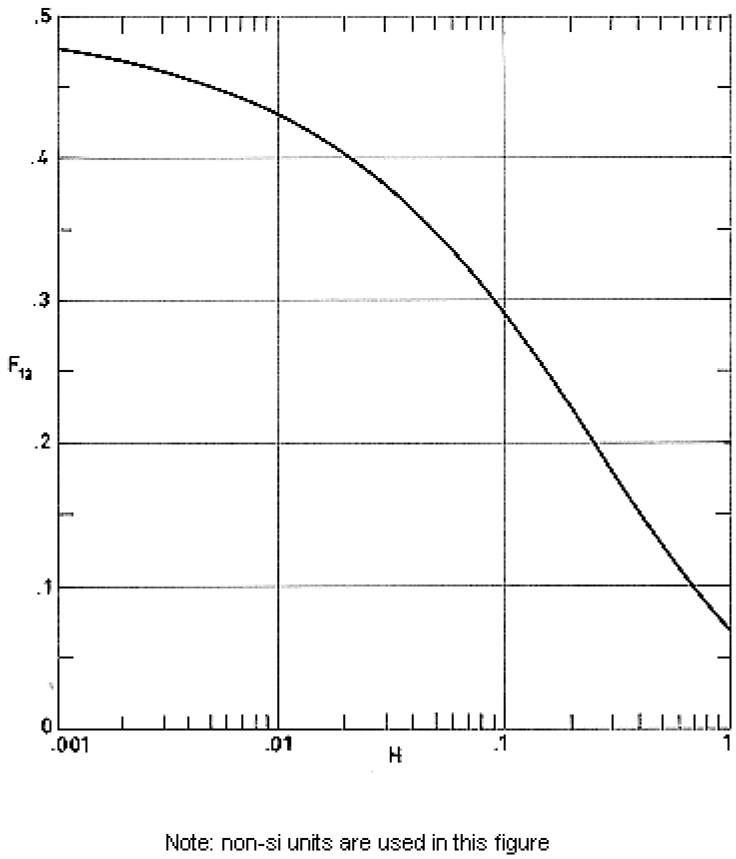
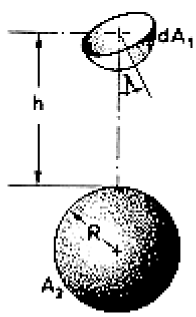


Figure 4-11: F_{12} as a function of H in the case of an infinitesimal sphere viewing a finite sphere. Calculated by the compiler.

4.2.5.2 Convex hemispherical surface to outer sphere

Convex surface of an infinitesimal hemisphere to sphere.

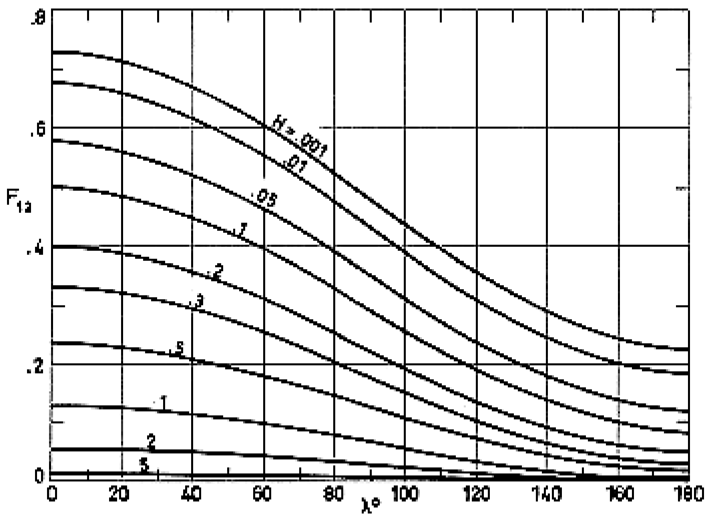
$H = h/R$



Formula:

$$F_{12} = \frac{1}{2}\left[1 - \frac{\sqrt{H^2 + 2H}}{1+H} + \frac{\cos\lambda}{2}\left(\frac{1}{1+H}\right)^2\right] \qquad [4\text{-}21]$$

Reference: Watts (1965) [50].



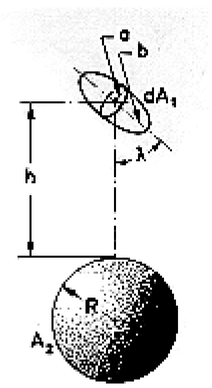
Note: non-si units are used in this figure

Figure 4-12: F_{12} as a function of angle λ for different values of the dimensionless distance H. Calculated by the compiler.

4.2.6 Ellipsoidal to spherical

Infinitesimal ellipsoid of revolution to sphere.

$H = h/R$
$A = a/b$



Formula:

$$F_{12} = \frac{1}{S}\int_0^{2\pi} d\phi \int_0^{\theta_o} \sqrt{\frac{A^4 + \tan^2\Delta}{A^2 + \tan^2\Delta}} \sin\left[\Delta + \tan^{-1}\left(A^2 \cot\Delta\right)\right] \sin\theta d\theta \qquad [4\text{-}22]$$

where

$$\sin\theta_o = \frac{1}{1+H} \qquad [4\text{-}23]$$

$$\cos\Delta = \cos\theta\cos\lambda + \sin\theta\sin\lambda\cos\phi \qquad [4\text{-}24]$$

$$S = 2\pi\left(A + \frac{\sin^{-1}\sqrt{1-A^2}}{\sqrt{1-A^2}}\right) \quad if\ A < 1(prolate\ ellipsoid) \qquad [4\text{-}25]$$

$$S = 2\pi\left[A + \frac{\ln\left(A+\sqrt{A^2-1}\right)}{\sqrt{A^2-1}}\right] \quad if\ A > 1(oblate\ ellipsoid) \qquad [4\text{-}26]$$

Reference: Watts (1965) [50].

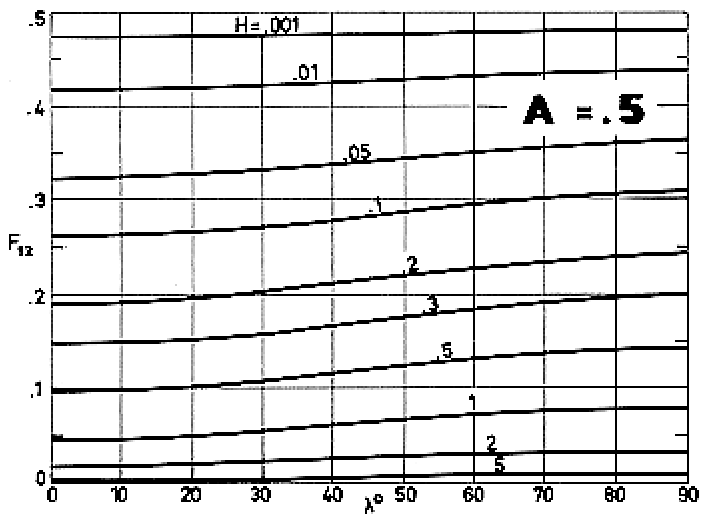


Figure 4-13: F_{12} as a function of λ and H, for A = 0,5. Calculated by the compiler.

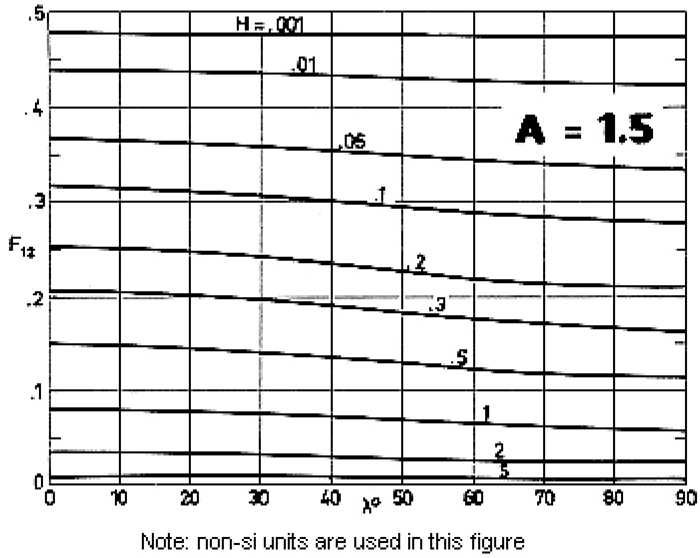


Figure 4-14: F_{12} as a function of λ and H, for A = 1,5. Calculated by the compiler.

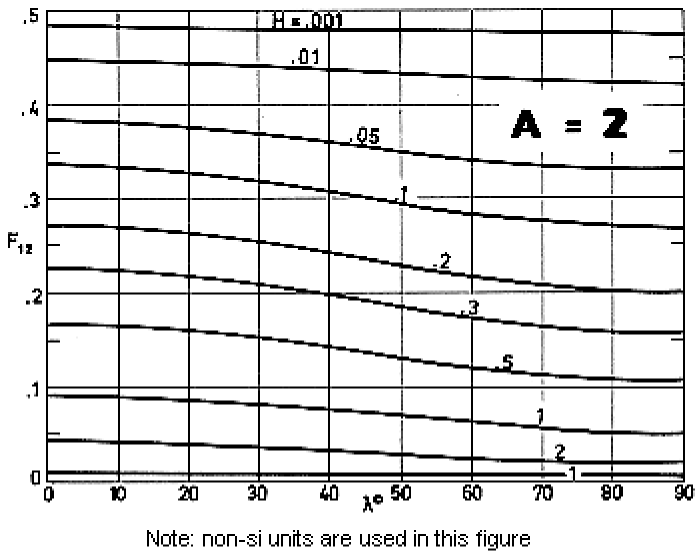
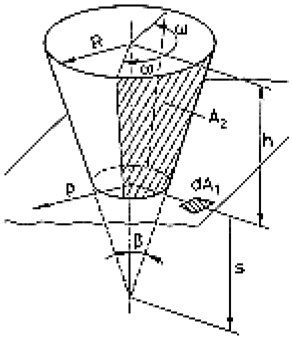


Figure 4-15: F_{12} as a function of λ and H, for A = 2. Calculated by the compiler.

4.2.7 Planar to conical

Infinitesimal surface of a ring to an inverted coaxial cone.

Two different configurations could arise depending on whether the cone intersects or not the planar surface.



$L = s/(h\text{-}s)$
$M = \rho/R$

$$\omega = 2\tan^{-1}\sqrt{\frac{M+L}{M-L}} \qquad [4\text{-}27]$$

Configuration 1. Cone and plane dA_1 intersect. ($s < 0$, $\rho \geq -s\tan\beta$).

Configuration 2. Cone and plane dA_1 do not intersect. ($s > 0$).

Formula:

Configuration 1:

$$F_{12} = \frac{\sin\beta}{\pi}\tan^{-1}\left(\frac{L+1}{\sin\beta\sqrt{M^2-L^2}}\right) + \frac{1}{\pi}\tan^{-1}\sqrt{\frac{M-L}{M+L}} + f(M,L,\beta) \qquad [4\text{-}28]$$

$$f(M,L,\beta) = \frac{(1-M^2)\tan^2\beta-(L+1)^2}{\pi\sqrt{\left[(1+M^2)\tan^2\beta+(L+1)^2\right]^2-4M^2\tan^4\beta}} \tan^{-1}\sqrt{\frac{M+L}{M-L}\frac{(1+M^2)\tan^2\beta+(L+1)^2}{(1-M^2)\tan^2\beta+(L+1)^2}} \qquad [4\text{-}29]$$

Configuration 2:

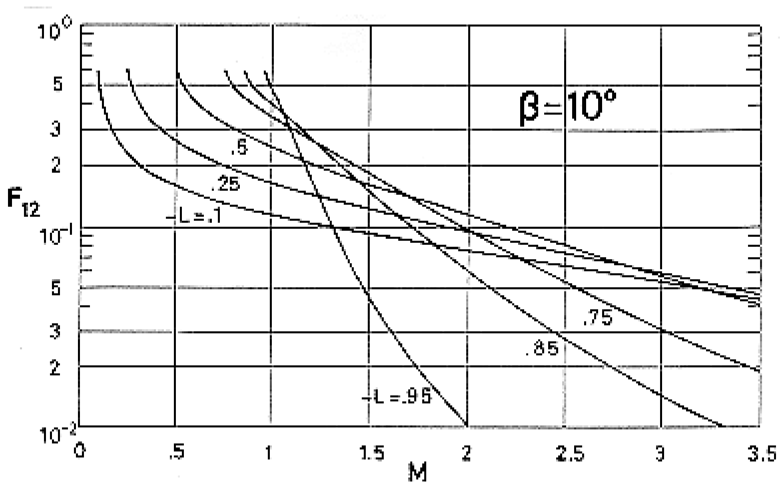
Case 2a: $M > L$

$$F_{12} = \frac{\omega}{2\pi} + \frac{\sin\beta}{\pi}\left[\tan^{-1}\left(\frac{L+1}{\sin\beta\sqrt{M^2-L^2}}\right) - \tan^{-1}\left(\frac{L}{\sin\beta\sqrt{M^2-L^2}}\right)\right] + f(M,L,\beta) \qquad [4\text{-}30]$$

Case 2b: $M \le L$ (Cone is seen as a disc of radius R. See the sketch).

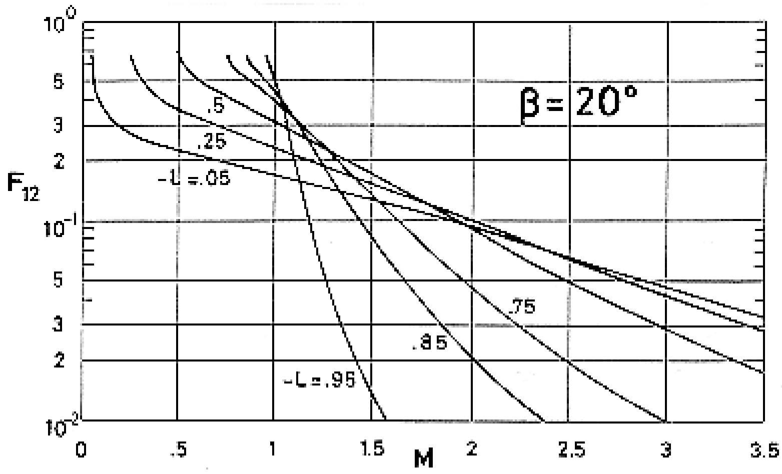
$$F_{12} = \frac{1}{2}\left[1 + \frac{(1-M^2)\tan^2\beta - (L+1)^2}{\pi\sqrt{\left[(1+M^2)\tan^2\beta + (L+1)^2\right]^2 - 4M^2\tan^4\beta}}\right] \qquad [4\text{-}31]$$

Reference: Minning (1977) [25].



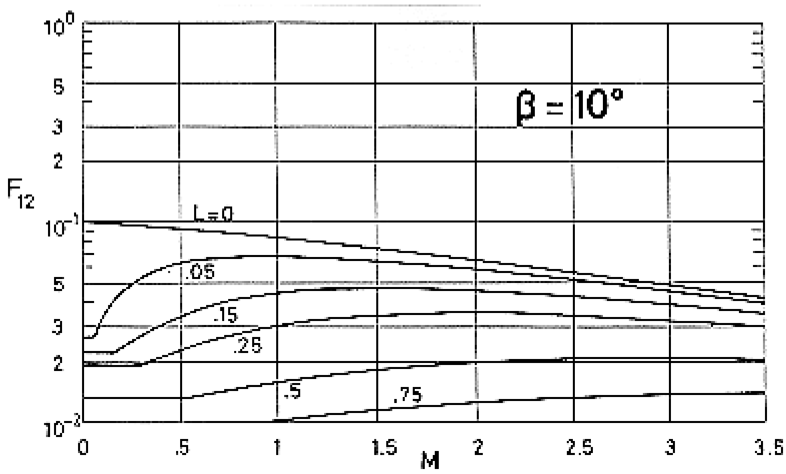
Note: non-si units are used in this figure

Figure 4-16: Values of F_{12} vs. M for different values of L. Configuration 1, β = 10°. Calculated by the compiler.



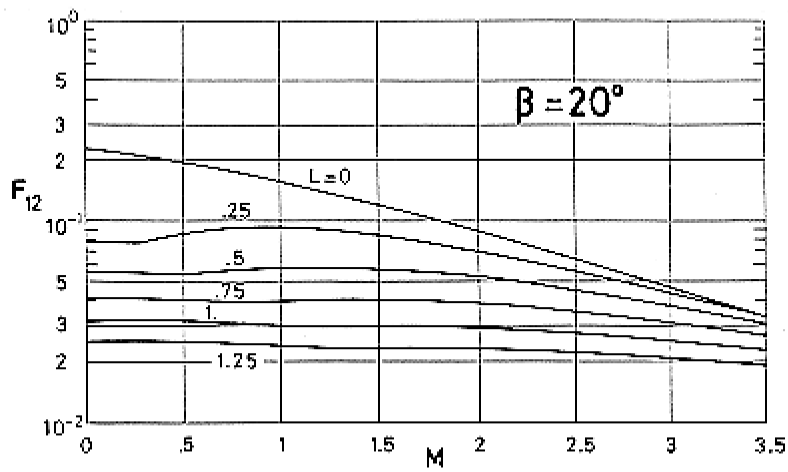
Note: non-si units are used in this figure

Figure 4-17: Values of F_{12} vs. M for different values of L. Configuration 1, β = 20°. Calculated by the compiler.



Note: non-si units are used in this figure

Figure 4-18: Values of F_{12} vs. M for different values of L. Configuration 2, β = 10°. Calculated by the compiler.



Note: non-si units are used in this figure

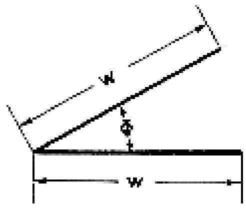
Figure 4-19: Values of F_{12} vs. M for different values of L. Configuration 2, β = 20°. Calculated by the compiler.

4.3 Finite to finite surface

4.3.1 Planar to planar. Two-dimensional configurations

4.3.1.1 Two strips of equal width at any angle

Two infinitely long plates of equal finite width *w*, having one common edge, and at an included angle ϕ to each other.



Formula:

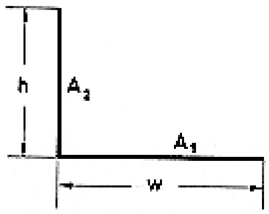
$$F_{12} = F_{21} = 1 - \sin\frac{\phi}{2} \qquad [4\text{-}32]$$

Reference: Siegel & Howell (1972) [37].

4.3.1.2 Two strips of unequal width normal to each other

Two infinitely long plates of unequal finite with, having one common edge, and at an included angle $\phi = 90°$ to each other.

$H = h/w$



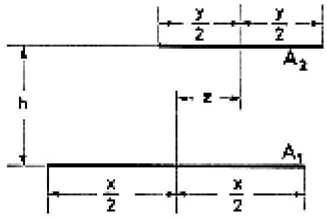
Formula:

$$F_{12} = HF_{21} = \frac{1}{2}\left[1 + H - \sqrt{1 + H^2}\right] \qquad [4\text{-}33]$$

References: Kreith (1962) [22], Siegel & Howell (1972) [37].

4.3.1.3 Two parallel strips

Two infinitely long parallel strips of unequal width



$X = x/h$
$Y = y/h$
$Z = z/h$

Formula:

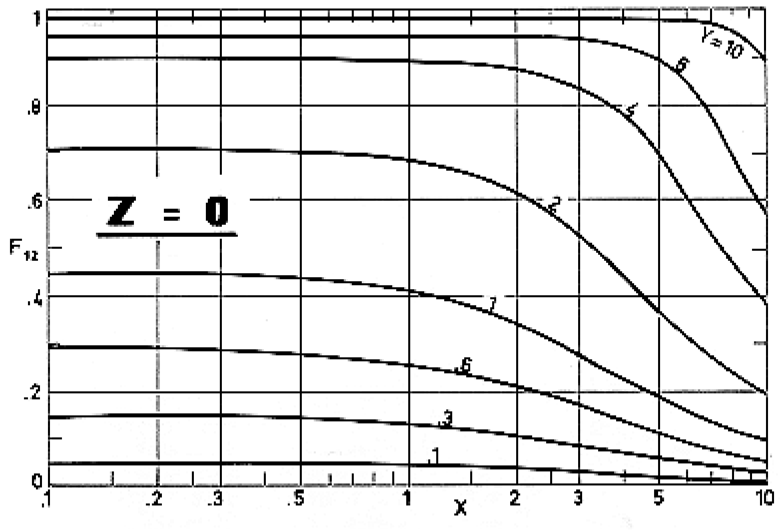
$$F_{12} = \frac{Y}{X} F_{21} = \frac{1}{2X} \left[\begin{array}{l} \sqrt{1 + \left(\frac{Y + X - 2Z}{2}\right)^2} + \sqrt{1 + \left(\frac{Y + X + 2Z}{2}\right)^2} - \\ - \sqrt{1 + \left(\frac{X - Y - 2Z}{2}\right)^2} - \sqrt{1 + \left(\frac{X - Y + 2Z}{2}\right)^2} \end{array} \right] \quad [4\text{-}34]$$

Reference: Kutateladze & Borishanskii (1966) [23].

When $X = Y$ and $Z = 0$, one obtains:

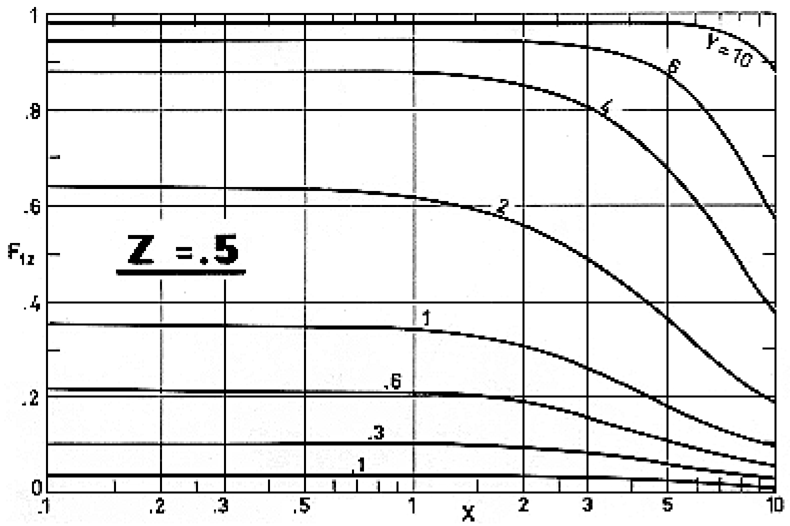
$$F_{12} = F_{21} = \frac{1}{X} \left[\sqrt{1 + X^2} - 1 \right] \quad [4\text{-}35]$$

References: Kreith (1962) [22], Siegel & Howell (1972) [37].



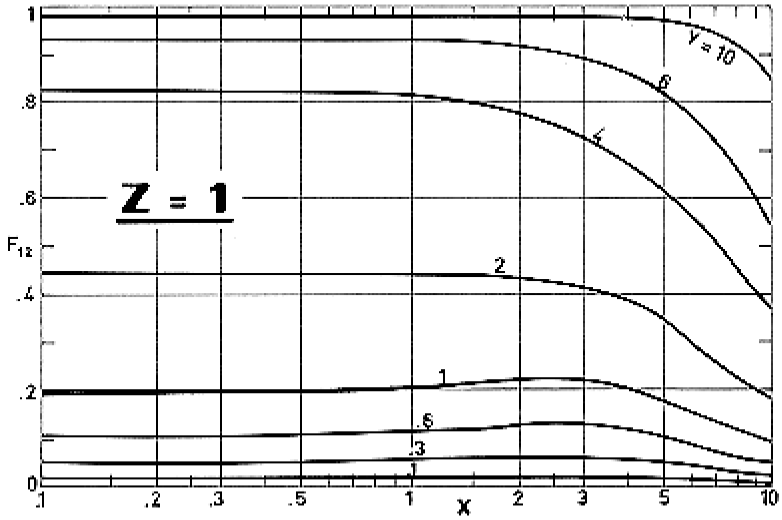
Note: non-si units are used in this figure

Figure 4-20: Values of F_{12} as a function of X and Y, for $Z = 0$. Calculated by the compiler.



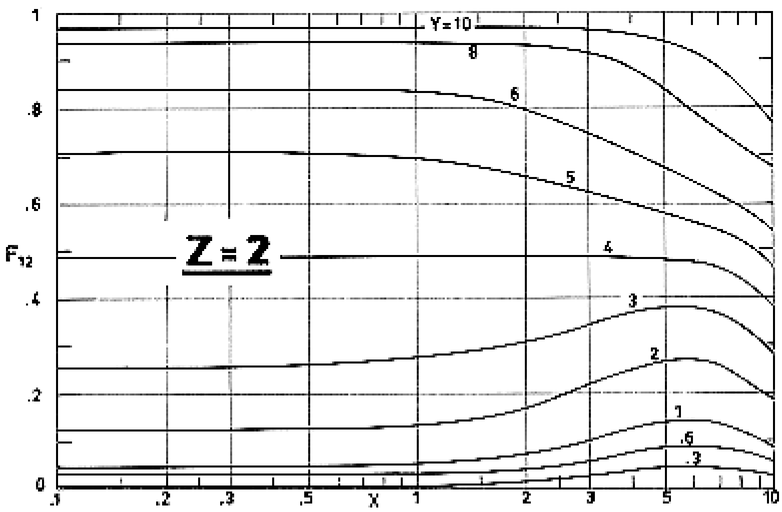
Note: non-si units are used in this figure

Figure 4-21: Values of F_{12} as a function of X and Y, for $Z = 0{,}5$. Calculated by the compiler.



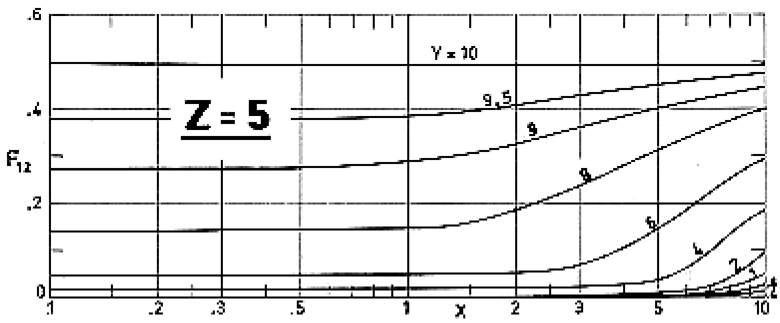
Note: non-si units are used in this figure

Figure 4-22: Values of F_{12} as a function of X and Y, for $Z = 1$. Calculated by the compiler.



Note: non-si units are used in this figure

Figure 4-23: Values of F_{12} as a function of X and Y, for $Z = 2$. Calculated by the compiler.



Note: non-si units are used in this figure

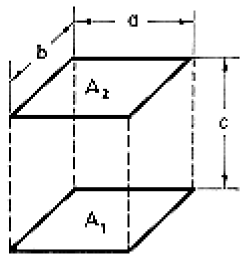
Figure 4-24: Values of F_{12} as a function of X and Y, for $Z = 5$. Calculated by the compiler.

4.3.2 Planar to planar. Three-dimensional configurations

4.3.2.1 Parallel rectangles of the same dimensions

Parallel, directly opposed rectangles of same width and length

$X = a/c$
$Y = b/c$

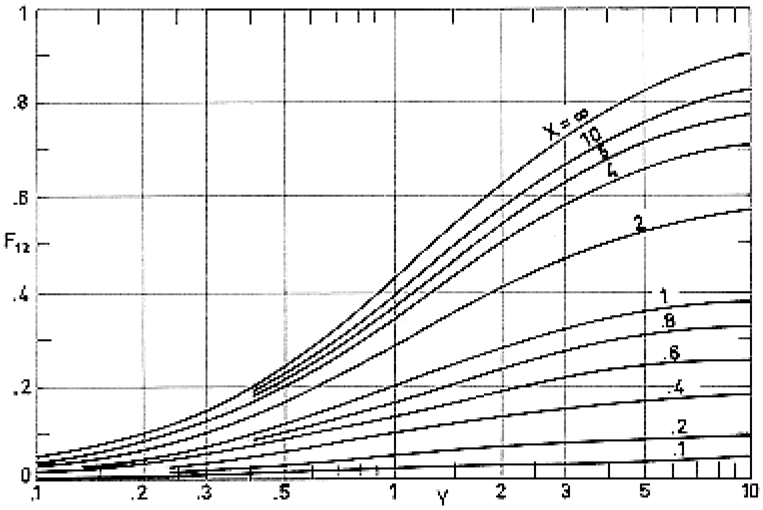


Formulae:

$$F_{12} = \frac{2}{\pi XY}\left\{ \begin{array}{l} \ln\left[\frac{(1+X^2)(1+Y^2)}{1+X^2+Y^2}\right]^{1/2} + X\sqrt{1+Y^2}\tan^{-1}\frac{X}{\sqrt{1+Y^2}} + \\ +Y\sqrt{1+X^2}\tan^{-1}\frac{Y}{\sqrt{1+X^2}} - X\tan^{-1}X - Y\tan^{-1}Y \end{array} \right\} \quad [4\text{-}36]$$

When $X \approx Y << 1$, $F_{12} = XY/\pi$.

References: Hamilton & Morgan (1952) [15], Jakob (1957) [19], Kreith (1962) [22], Hsu (1967) [18], Siegel & Howell (1972) [37].



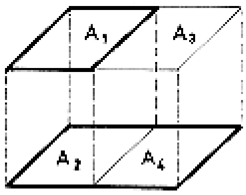
Note: non-si units are used in this figure

Figure 4-25: Values of F_{12} as a function of X and Y. Calculated by the compiler.

4.3.2.2 Parallel rectangle of unequal dimensions

The following view factors can be deduced by using the results for two parallel directly opposed rectangles.

Two rectangles in parallel planes with one rectangle directly opposite to portion of the other.

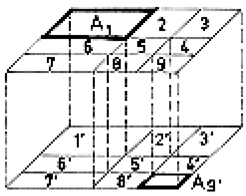


View factor is:

$$F_{1(2,4)} = \frac{1}{2A_1}\left[A_{(1,3)}F_{(1,3)(2,4)} + A_1F_{12} - A_3F_{34}\right] \quad [4\text{-}37]$$

References: Moon (1961) [26], Kreith (1962) [22], Hsu (1967) [18].

Two rectangles of arbitrary size in parallel planes with one edge of a rectangle parallel to one of the other.



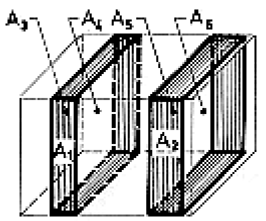
View factor is:

$$A_1 F_{19'} = \frac{1}{4} \begin{bmatrix} K_{(1,2,3,4,5,6,7,8,9)^2} - K_{(1,2,5,6,7,8)^2} - K_{(2,3,4,5,8,9)^2} - K_{(1,2,3,4,5,6,7,8,9)^2} - \\ - K_{(1,2,3,4,5,6)^2} - K_{(1,2,5,6)^2} - K_{(2,3,4,5)^2} - K_{(4,5,8,9)^2} - K_{(4,5)^2} - K_{(5,8)^2} - \\ - K_{(5,6)^2} - K_{(4,5,6,7,8,9)^2} - K_{(5,6,7,8)^2} - K_{(4,5,6)^2} - K_{(2,5,8)^2} - K_{(2,5)^2} - K_{5^2} \end{bmatrix} \quad [4\text{-}38]$$

where $K_{m2} = A_m F_{mm'}$.

Reference: Hsu (1967) [18].

Finite area on interior of a rectangular channel.



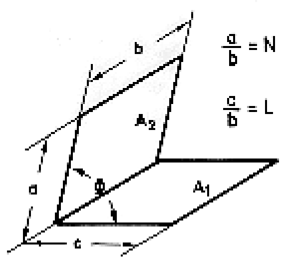
View factor is:

$$F_{12} = \frac{A_4}{A_1}\left(F_{45} + F_{36} + F_{46} - F_{35}\right) \quad [4\text{-}39]$$

Reference: Siegel & Howell (1972) [37].

4.3.2.3 Rectangles with one common edge

Two rectangles A_1 and A_2 with one common edge and included angle ϕ between the two planes.



Formulae:

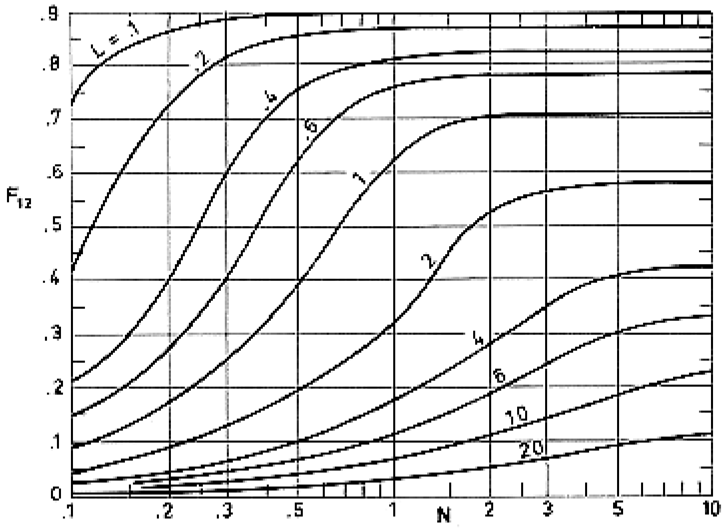
$$F_{12}=\frac{1}{\pi L}\left(\begin{array}{l} -\frac{\sin 2\Phi}{4}\left[NL\sin\Phi+\left(\frac{\pi}{2}-\Phi\right)\left(N^2+L^2\right)+L^2\tan^{-1}\frac{N-L\cos\Phi}{L\sin\Phi}+N^2\tan^{-1}\frac{L-N\cos\Phi}{N\sin\Phi}\right]+ \\ +\frac{\sin^2\Phi}{4}\log_e\left\{\left[\frac{\left(1+N^2\right)\left(1+L^2\right)}{1+N^2+L^2-2NL\cos\Phi}\right]^{\cos ec^2\Phi+\cot^2\Phi}\left[\frac{L^2\left(1+N^2+L^2-2NL\cos\Phi\right)}{\left(1+L^2\right)\left(N^2+L^2-2NL\cos\Phi\right)}\right]^{L^2}\right\}+ \\ +\frac{N^2\sin^2\Phi}{4}\log_e\left[\left(\frac{N^2}{N^2+L^2-2NL\cos\Phi}\right)\left(\frac{1+N^2}{1+N^2+L^2-2NL\cos\Phi}\right)^{\cos 2\Phi}\right]+ \\ +L\tan^{-1}\frac{1}{L}+N\tan^{-1}\frac{1}{N}-\sqrt{N^2+L^2-2NL\cos\Phi}\,\cot^{-1}\sqrt{N^2+L^2-2NL\cos\Phi}+ \\ +\frac{N\sin\Phi\sin 2\Phi}{2}\sqrt{1+N^2\sin^2\Phi}\left[\tan^{-1}\frac{N\cos\Phi}{\sqrt{1+N^2\sin^2\Phi}}+\tan^{-1}\frac{L-N\cos\Phi}{\sqrt{1+N^2\sin^2\Phi}}\right]+ \\ +\cos\Phi\int_0^L\sqrt{1+z^2\sin^2\Phi}\left[\tan^{-1}\frac{N-z\cos\Phi}{\sqrt{1+z^2\sin^2\Phi}}+\tan^{-1}\frac{z\cos\Phi}{\sqrt{1+z^2\sin^2\Phi}}\right]dz \end{array}\right) \quad [4\text{-}40]$$

For $\Phi = 90°$,

$$F_{12}=\frac{1}{\pi L}\left(\begin{array}{l} L\tan^{-1}\frac{1}{L}+N\tan^{-1}\frac{1}{N}-\sqrt{N^2+L^2}\tan^{-1}\frac{1}{\sqrt{N^2+L^2}}+ \\ +\frac{1}{4}\log_e\left\{\left[\frac{\left(1+N^2\right)\left(1+L^2\right)}{1+N^2+L^2}\right]\left[\frac{L^2\left(1+N^2+L^2\right)}{\left(1+L^2\right)\left(N^2+L^2\right)}\right]^{L^2}\left[\frac{N^2\left(1+N^2+L^2\right)}{\left(1+N^2\right)\left(N^2+L^2\right)}\right]^{N^2}\right\} \end{array}\right) \quad [4\text{-}41]$$

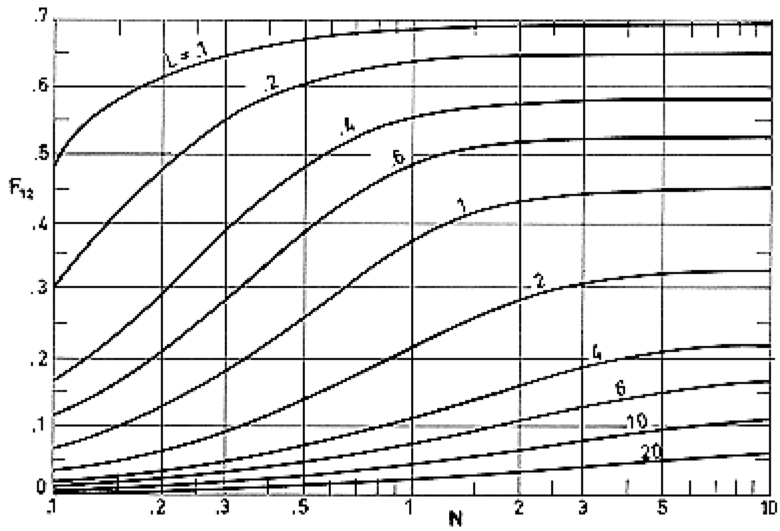
$$\lim_{L\to\infty}F_{12}=0 \quad , \quad \lim_{N\to\infty}F_{12}=\frac{1}{\pi}\left[\tan^{-1}\frac{1}{L}+\frac{1}{4L}\log_e\left(1+L^2\right)-\frac{L}{4}\log_e\frac{1+L^2}{L^2}\right] \quad [4\text{-}42]$$

References: Hamilton & Morgan (1952) [15], Hottel (1954) [17], Jakob (1957) [19], Kreith (1962) [22], Feingold (1966) [11], Redor (1973) [34].



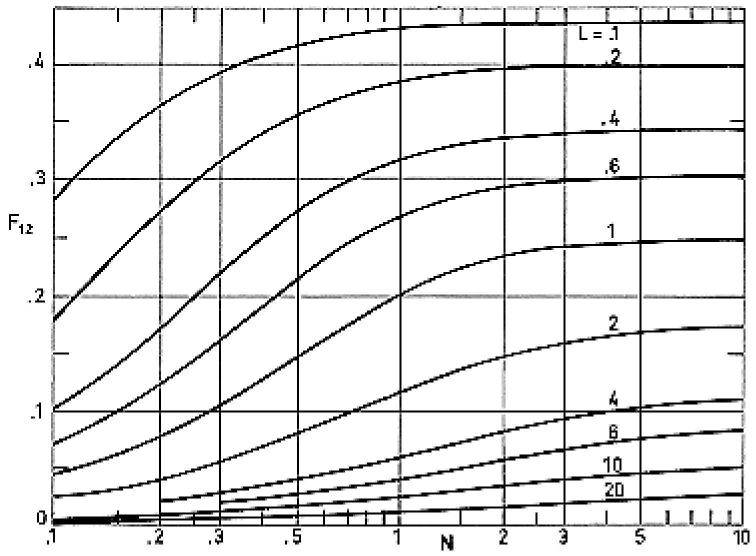
Note: non-si units are used in this figure

Figure 4-26: F_{12} as a function of L and N for $\Phi = 30°$. Table from Feingold (1966) [11], figure from Hamilton & Morgan (1952) [15].



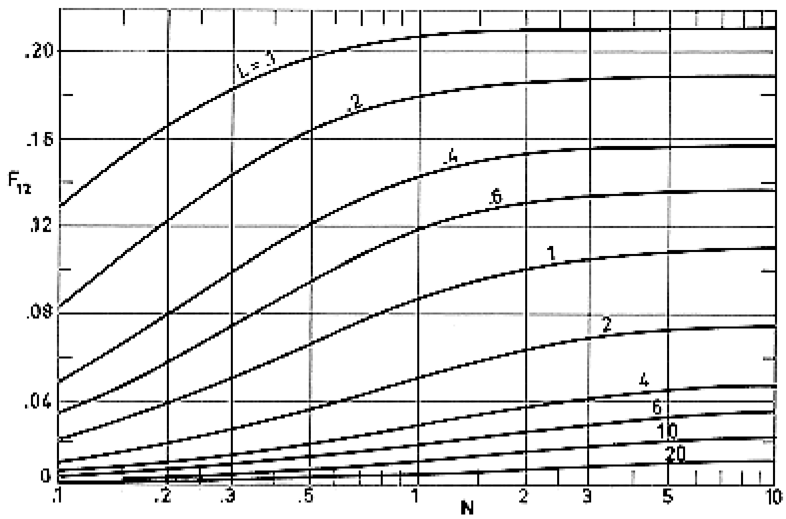
Note: non-si units are used in this figure

Figure 4-27: F_{12} as a function of L and N for $\Phi = 60°$. Table from Feingold (1966) [11], figure from Hamilton & Morgan (1952) [15].



Note: non-si units are used in this figure

Figure 4-28: F_{12} as a function of *L* and *N* for $\Phi = 90°$. Table from Feingold (1966) [11], figure from Hamilton & Morgan (1952) [15].



Note: non-si units are used in this figure

Figure 4-29: F_{12} as a function of *L* and *N* for $\Phi = 120°$. Table from Feingold (1966) [11], figure from Hamilton & Morgan (1952) [15].

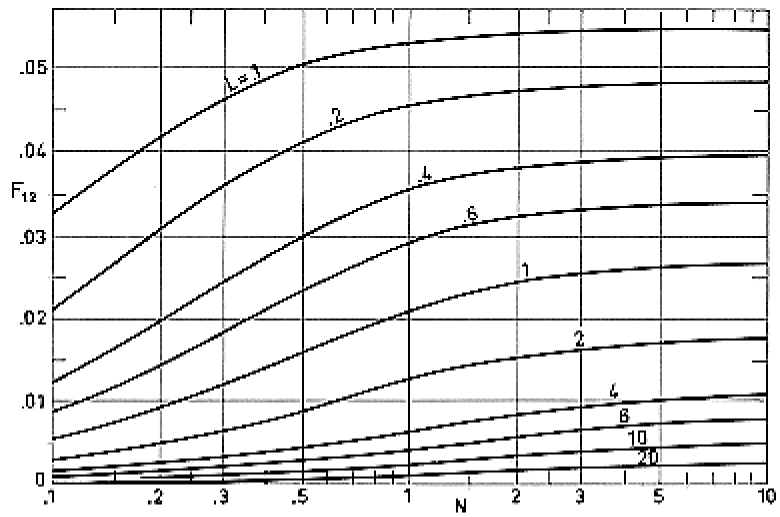
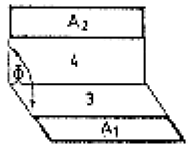


Figure 4-30: F_{12} as a function of L and N for Φ = 150°. Table from Feingold (1966) [11], figure from Hamilton & Morgan (1952) [15].

4.3.2.4 Rectangles placed in intersecting planes

The following view factors can be deduced by using the results for two rectangles with one common edge.

Two rectangles A_1 and A_2, with one side of A_1 parallel to one side of A_2 in planes intersecting at angle Φ.

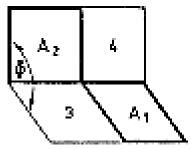


View Factor is:

$$F_{12} = \frac{A_{(1,3)}F_{(1,3)(2,4)} + A_3 F_{34} - A_3 F_{3(2,4)} - A_{(1,3)}F_{(1,3)4}}{A_1} \quad [4\text{-}43]$$

References: Kreith (1962) [22].

Two rectangles A_1 and A_2, with one side of A_1 parallel to one side of A_2, and one corner of A_1 touches a corner of A_2. Both planes intersect at an angle Φ.

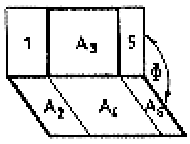


View Factor is:

$$F_{12} = \frac{A_{(1,3)}F_{(1,3)(2,4)} - A_1F_{14} - A_3F_{32}}{2A_1} \qquad [4\text{-}44]$$

References: Kreith (1962) [22].

Two rectangles A_1 and $A_{(2,4,6)}$, with one common edge and included angle Φ between the two planes.

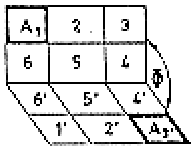


View Factor is:

$$F_{3(2,4,6)} = \frac{A_{(1,3)}F_{(1,3)(2,4)} + A_{(3,5)}F_{(3,5)(4,6)} - A_1F_{12} - A_5F_{56}}{2A_3} \qquad [4\text{-}45]$$

References: Kreith (1962) [22].

Two rectangles A_1 and A_3'. One side of A_1 parallel to one side of A_3'. Both planes intersect at angle Φ.



View Factor is:

$$A_1F_{13'} = \frac{1}{2}\begin{bmatrix} K_{(1,2,3,4,5,6)^2} - K_{(2,3,4,5)^2} - K_{(1,2,5,6)^2} + K_{(4,5,6)^2} - K_{(1,2,3,4,5,6)(4',5',6')} - \\ - K_{(4,5,6)(1',2',3',4',5',6')} + K_{(1,2,5,6)(5',6')}K_{(1,2,3,4,5)(4',5')} + K_{(5,6)(1',2',5',6')} + \\ + K_{(4,5)(2',3',4',5')} + K_{(2,5)^2} - K_{(2,5)5'} - K_{(5,6)^2} - K_{(4,5)^2} - K_{5(2',5')} - K_{5^2} \end{bmatrix} \qquad [4\text{-}46]$$

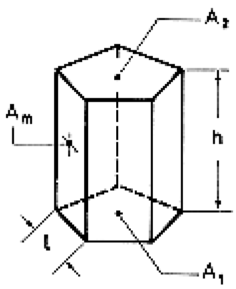
where $K_{(m)(n)} = A_mF_{mn}$ and $K_m2 = A_mF_{mm'}$.

References: Hamilton & Morgan (1952) [15], Kreith (1962) [22].

4.3.2.5 Regular polygons forming the bases of a prism

Two parallel regular polygons forming the bases of a right prism.

$L = l/h$



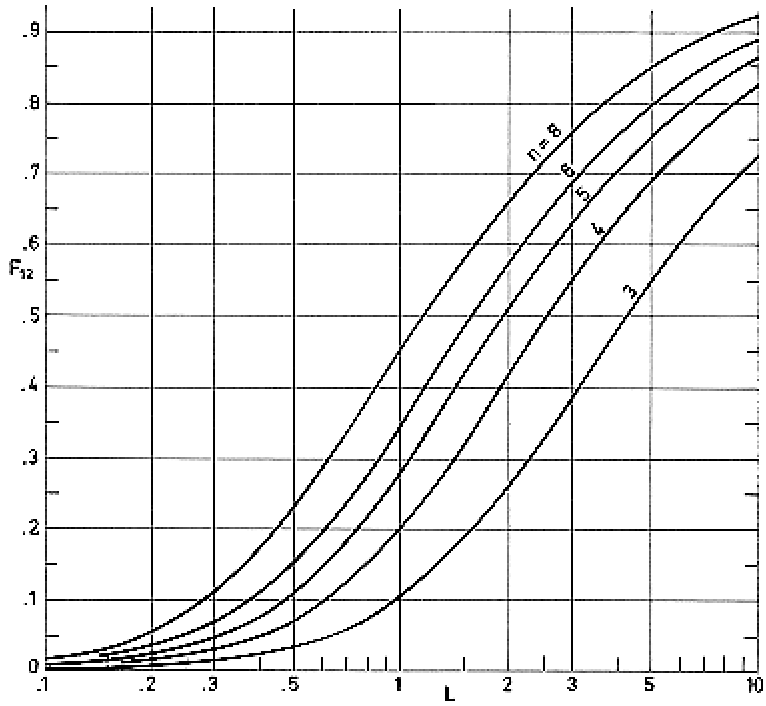
Formula:

$$F_{12} = 1 - \sum_{m=3}^{n+2} F_{1m} \qquad [4\text{-}47]$$

where n is the number of sides of the polygon.

The values of F_{1m} can be deduced by using the results corresponding to two rectangles with one common edge, by applying simple factor algebra.

Reference: Feingold (1966) [11].



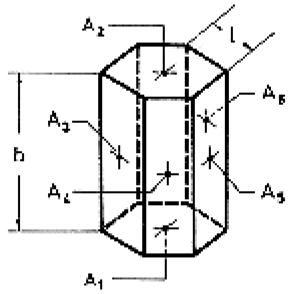
Note: non-si units are used in this figure

Figure 4-31: Values of F_{12} as a function of L for different regular polygons. n is the number of sides of the polygon. From Feingold (1966) [11].

4.3.2.6 Several areas of a prismatic configuration

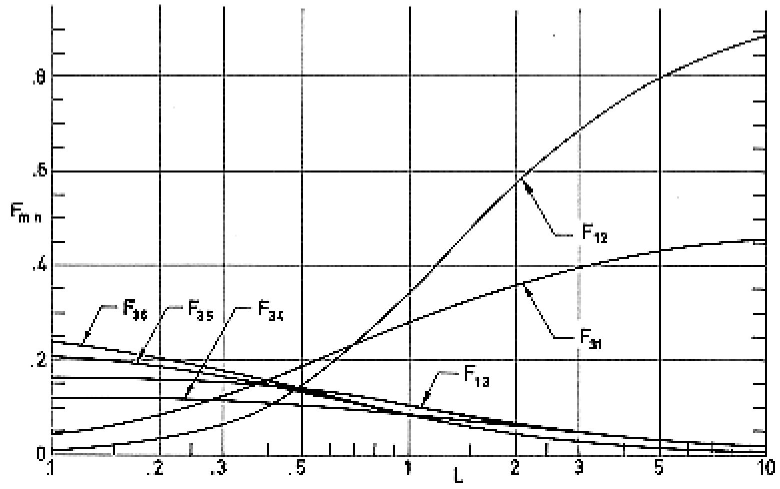
View factors between various areas in a honeycomb structure.

$L = l/h$



The results being presented have been obtained by combination of the data on two parallel regular polygons with those concerning two rectangles with one common edge.

Reference: Feingold (1966) [11].

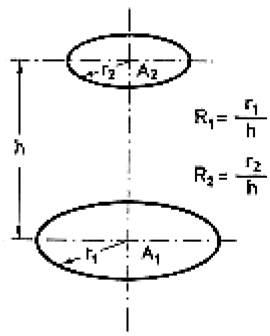


Note: non-si units are used in this figure

Figure 4-32: View factors between different faces of a honeycomb cell as a function of the cell length, *L*. From Feingold (1966) [11].

4.3.2.7 Parallel coaxial discs

Parallel circular discs with centers along the same normal.



Formula:

$$F_{12} = \frac{1}{2}\left[x - \sqrt{x^2 - 4\left(\frac{R_2}{R_1}\right)^2}\right] \quad [4\text{-}48]$$

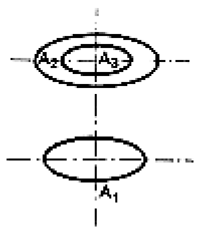
with

$$x = 1 + \frac{1 + R_2^2}{R_1^2} \quad [4\text{-}49]$$

References: Hamilton & Morgan (1952) [15], Leuenberger & Person (1956) [24], Jakob (1957) [19], Eckert & Drake (1959) [9], Kreith (1962) [22], Siegel & Howell (1972) [37].

Comments: The following view factors may be deduced from the previous one.

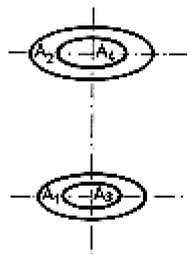
Directly opposed ring to disc of arbitrary radii.



$F_{12} = F_{1(2,3)} - F_{13}$

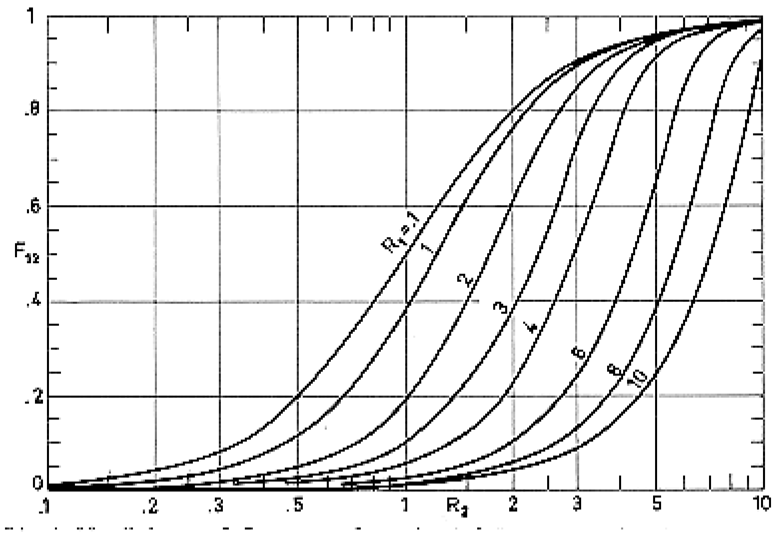
Reference: Leuenberger & Person (1956) [24].

Parallel, directly opposed plane ring areas



$$F_{12} = \left[1 + \frac{A_3}{A_1}\right]\left[F_{(1,3)(2,4)} - F_{(1,3)4}\right] - \frac{A_3}{A_1}\left[F_{3(2,4)} - F_{34}\right] \quad [4\text{-}50]$$

References: Hamilton & Morgan (1952) [15], Leuenberger & Person (1956) [24], Siegel & Howell (1972) [37].



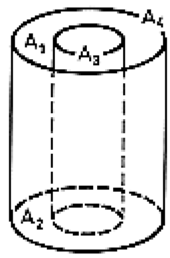
Note: non-si units are used in this figure

Figure 4-33: Values of F_{12} as a function of R_1 and R_2 in the case of two parallel coaxial discs. Calculated by the compiler.

4.3.2.8 Rings at opposite ends of a circular cylinder

The following view factor may be obtained from those corresponding to two coaxial cylinders of equal length.

annular ring to an equal annular ring placed at the opposite end of the cylinder. The view factor is:



Formula:

$$F_{12} = 1 - \frac{A_3}{2A_1}\left[1 - \frac{A_4}{A_3}\left(F_{44} + 2F_{43} - 1\right)\right] \qquad [4\text{-}51]$$

For F_{43} and F_{44} see clause 4.3.8.1.

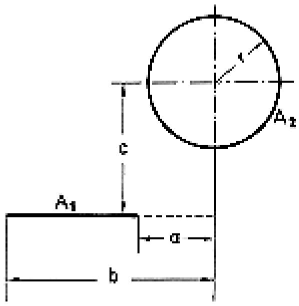
References: Leuenberger & Person (1956) [24]; Sparrow, Miller & Jonsson (1962) [43].

4.3.3 Planar to cylindrical. Two-dimensional configurations

4.3.3.1 Plane to circular cylinder

Infinitely long plane of finite width, to parallel infinitely long cylinder.

$c \geq r$



Formula:

$$F_{12} = \frac{r}{b-a}\left[\tan^{-1}\frac{b}{c} - \tan^{-1}\frac{a}{c}\right] \qquad [4\text{-}52]$$

where the range of $\tan^{-1}x$ is $-p/2$ to $p/2$.

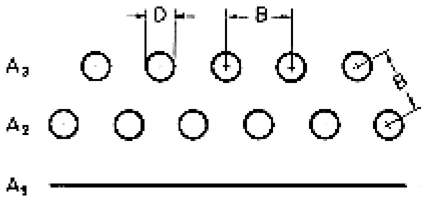
References: Hamilton & Morgan (1952) [15], Kreith (1962) [22], Feingold & Gupta (1970) [12], Siegel & Howell (1972) [37].

Comments: The expressions given by Hamilton & Morgan (1952) [15] and Kreith (1962) [22] are in error as has been pointed out both by Feingold & Gupta (1970) [12], and Siegel & Howell (1972) [37].

4.3.3.2 Plane to rows of circular cylinders

Infinite plane to first row and second row of an infinite number of parallel staggered tubes having equal diameters.

$K = B/D$

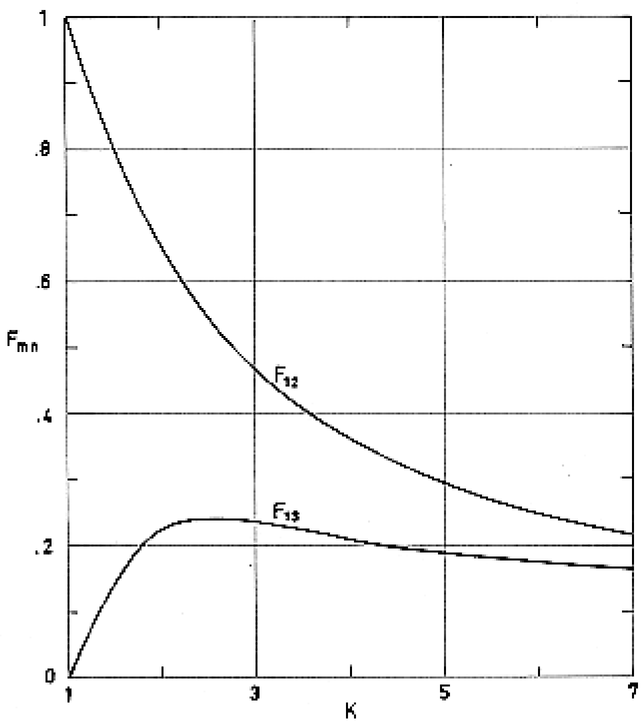


Formula:

$$F_{12} = \frac{K + \tan^{-1}\sqrt{K^2-1} - \sqrt{K^2-1}}{K} \qquad [4\text{-}53]$$

F_{13} has been obtained graphically. No analytical expression is available.

References: Hottel (1954) [17], Jacob (1957) [19].



Note: non-si units are used in this figure

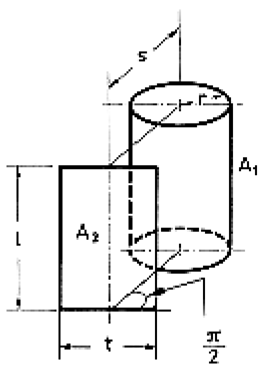
Figure 4-34: Values of F_{12} and F_{13} as a function of the parameter *K*. From Jakob (1957) [19].

4.3.4 Planar to cylindrical. three-dimensional configurations

4.3.4.1 Finite length cylinder to outer rectangle

Finite length cylinder to rectangle with two edges parallel to cylinder axis and length equal cylinder.

$R = r/l$
$Z = s/r$
$T = t/r$



Formula:

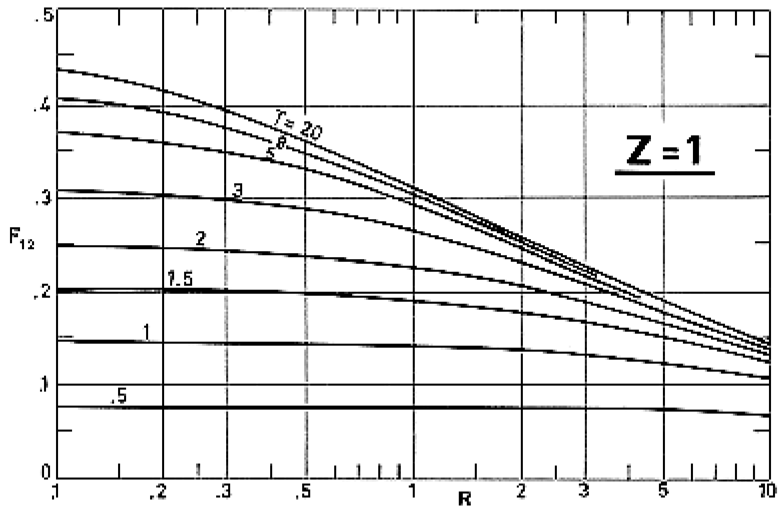
$$F_{12} = \frac{T}{2\pi}\int_0^1 Zv^2\left(1-\frac{1}{\pi}\left\{\cos^{-1}\frac{1+y}{1-y}-\frac{1}{2R}\left[\begin{array}{l}\sqrt{(1-y)^2+4R^2}\cos^{-1}\left(v\frac{1+y}{1-y}\right)+\\ +(1+y)\sin^{-1}v-\frac{\pi}{2}(1-y)\end{array}\right]\right\}\right) \qquad [4\text{-}54]$$

where:

$$y = R^2\left(1-Z^2-\frac{T^2x^2}{4}\right) \qquad [4\text{-}55]$$

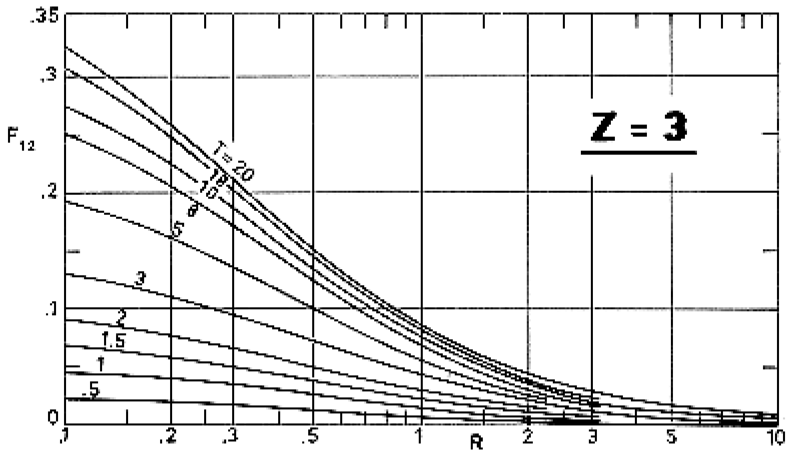
$$v = \frac{1}{\sqrt{Z^2+\frac{T^2x^2}{4}}} \qquad [4\text{-}56]$$

Reference: Leuenberger & Person (1956) [24].



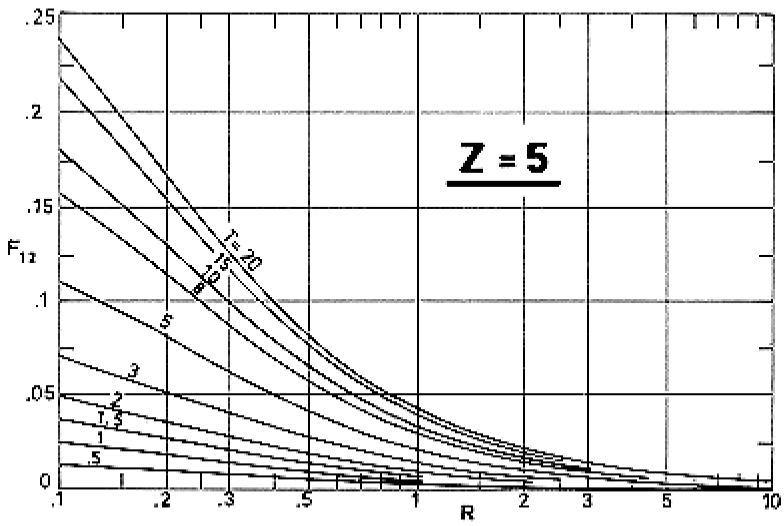
Note: non-si units are used in this figure

Figure 4-35: F_{12} as a function of T and R. Calculated by the compiler.



Note: non-si units are used in this figure

Figure 4-36: *F12* as a function of *T* and *R*. Calculated by the compiler.



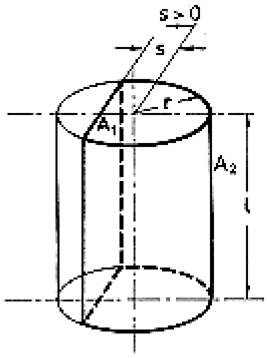
Note: non-si units are used in this figure

Figure 4-37: *F12* as a function of *T* and *R*. Calculated by the compiler.

4.3.4.2 Inner rectangle to finite-length cylinder

Cylinder and plane of equal length parallel to cylinder axis, plane inside cylinder.

$R = r/l$
$Z = s/r$



Formula:

$$F_{12} = 1 - \frac{1}{\pi}\left[2\tan^{-1}2A - \frac{1}{2A}\ln\left(1+4A^2\right)\right] - \frac{1}{4\pi A}\int_0^A \left(x\left[1\left(A^2 - x^2\right)\ln\frac{A+x}{A-x} + \left(1 + A^2 - x^2\right)\ln\frac{(A-x)^2+1}{(A+x)^2+1}\right] + RZ\left\{ \begin{array}{l} 2\left[\left(1 - A^2 + x^2\right)\cos^{-1}Z - \pi\right] + \\ + \sqrt{\left[1 + R^2\left(1+Z^2\right) + x^2\right]^2 - 4R^2\left(R^2Z^2 + x^2\right)} \\ \left[\begin{array}{l} \cos^{-1}\left(\frac{Z}{|Z|}\frac{\left(R^2Z^2 + xA\right)(A+x)^2 - R^2Z^2 + xA}{R\sqrt{R^2Z^2+x^2}\left[(A+x)^2+1\right]}\right) + \\ + \cos^{-1}\left(\frac{Z}{|Z|}\frac{\left(R^2Z^2 - xA\right)(A-x)^2 - R^2Z^2 - xA}{R\sqrt{R^2Z^2+x^2}\left[(A-x)^2+1\right]}\right) \end{array}\right] \end{array} \right\} \right) \left(R^2Z^2 + x^2\right)^{-1} dx \quad [4\text{-}57]$$

where:

$$A = R\sqrt{1 - Z^2} \quad [4\text{-}58]$$

and, for any argument x:

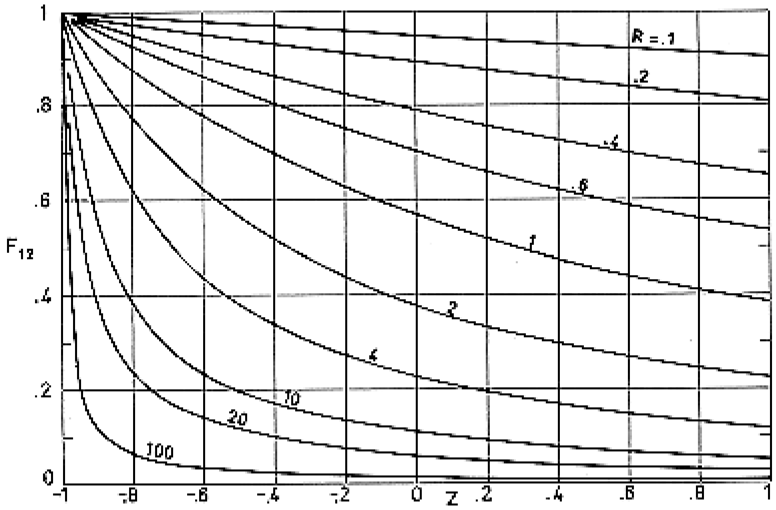
$$-\frac{\pi}{2} < \tan^{-1}\xi < \frac{\pi}{2} \quad [4\text{-}59]$$

$$0 < \cos^{-1}\xi < \pi \quad [4\text{-}60]$$

$$\lim_{Z\to 1} F_{12} = 1 - \frac{1}{2R}\left(\sqrt{1+4R^2} - 1\right) \qquad [4\text{-}61]$$

$$\lim_{Z\to -1} F_{12} = 1 \qquad [4\text{-}62]$$

Reference: These expressions have been obtained by the compiler after Leuenberger & Person (1956) [24].

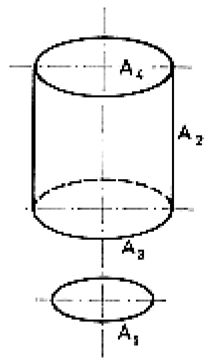


Note: non-si units are used in this figure

Figure 4-38: F_{12} as a function of Z, for different values of the dimensionless radius R. Calculated by the compiler.

4.3.4.3 Disc to inner surface of a coaxial cylinder

The following view factors may be obtained by use of those for two parallel circular discs with centers along the same normal.



Disk to the interior surface of a coaxial cylinder of larger or equal radius. The view factor is given by

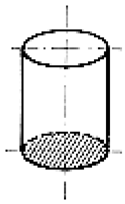
$F_{12} = F_{13} - F_{14}$

For F_{13} and F_{14} see clause 4.3.2.7.

Reference: Siegel & Howell (1972) [37]

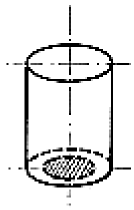
Particular cases of the previous configuration are:

Entire inner wall of finite cylinder to ends.



Reference: Bien (1966) [2].

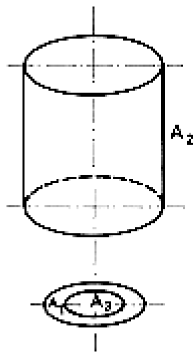
Inner surface of cylinder to disc at one end.



Reference: Leuenberger & Person (1956) [24].

4.3.4.4 Ring to inner surface of a coaxial cylinder

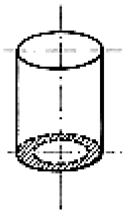
The following view factors may be obtained by use of that foe a disc viewing the inner surface of a coaxial cylinder of larger or equal radius.



Ring to the inner surface of a coaxial cylinder of larger or equal radius. The view factor is given by

$$F_{12} = \left[1 + \frac{A_3}{A_1}\right] F_{(1,3)2} - \frac{A_3}{A_1} F_{32} \qquad [4\text{-}63]$$

A particular case of the previous configuration is

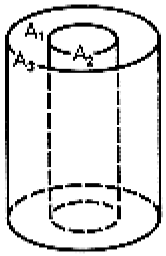


Inner surface of cylinder to annulus on one end.

Reference: Leuenberger & Person (1956) [24].

4.3.4.5 Finite-length coaxial cylinder to enclosed base

The following view factors may be obtained from those for two coaxial cylinder of equal length.



Inner or outer coaxial cylinders of the same length to annular ring placed at one end of the cylinders. The view factors are:

$F_{21} = (1 - F_{23})/2$

$F_{31} = (1 - F_{32} - F_{33})/2$

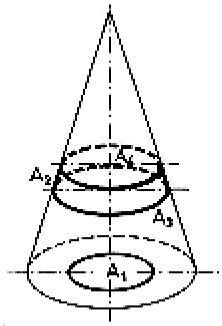
For F_{32} and F_{33} see clause 4.3.8.1.

References: Leuenberger & Person (1956) [24]; Sparrow, Miller & Jonsson (1962) [43].

4.3.5 Planar to conical

The following view factors may be obtained by use of those for two parallel circular discs.

Disc on the base of a right circular cone to an axisymmetrical portion of the conical surface.

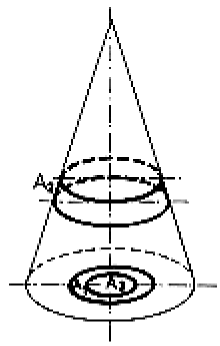


$F_{12} = F_{13} - F_{14}$

For F_{13} and F_{14} see clause 4.3.2.7.

Reference: Buschman & Pittman (1961) [3].

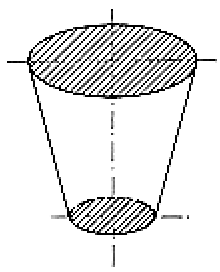
Ring on the base of a right circular cone to an axisymmetrical portion of the conical surface.



$$F_{12} = \left[1 + \frac{A_3}{A_1}\right] F_{(1,3)2} - \frac{A_3}{A_1} F_{32} \qquad [4\text{-}64]$$

Reference: Buschman & Pittman (1961) [3].

The following particular view factor can be obtained from the first one.



Inner surface of frustum of cone to ends.

References: Buschman & Pittman (1961) [3], Bien (1966) [2].

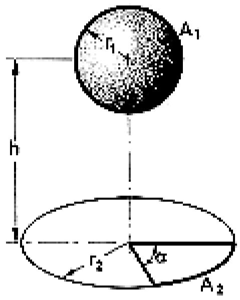
4.3.6 Spherical to planar

4.3.6.1 Sphere to sector of a coaxial disc

Sphere to sector of disc; normal to center of disc passes through center of sphere.

$R_2 = r_2/h$

$h \geq r_1$



Formula:

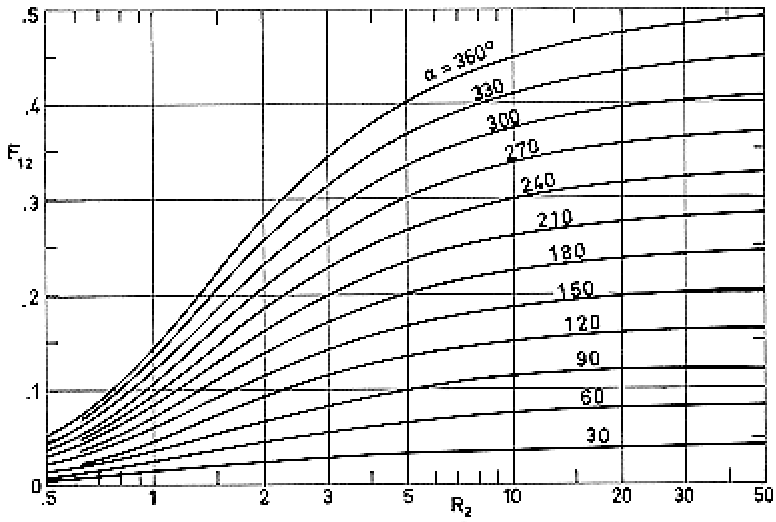
$$F_{12} = \frac{\alpha}{4\pi}\left[1 - \frac{1}{\sqrt{1+R_2^2}}\right] \qquad [4\text{-}65]$$

For $\alpha = 2\pi$

$$F_{12} = \frac{1}{2}\left[1 - \frac{1}{\sqrt{1+R_2^2}}\right] \qquad [4\text{-}66]$$

Comments: In the first of these two expressions α is measured in radians, although it is given in degrees in table and figure on the next page.

References: Feingold & Gupta (1970) [12], Siegel & Howell (1972) [37].



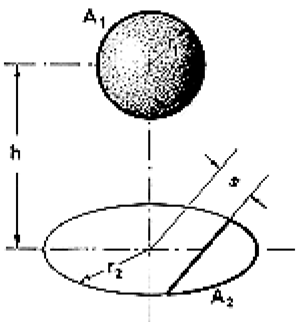
Note: non-si units are used in this figure

Figure 4-39: F_{12} as a function of R_2 for different values of the sector central angel α. Calculated by the compiler.

4.3.6.2 Sphere to segment of a coaxial disc

Sphere to segment of disc; normal to center of disc passes through center of sphere.

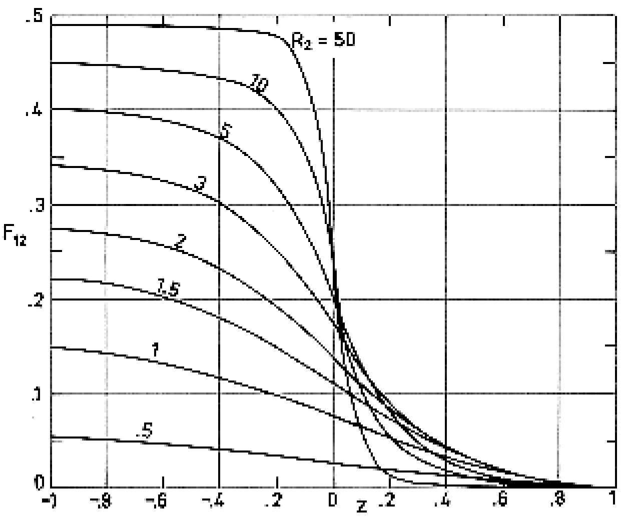
$R_2 = r_2/h$
$Z = s/r_2$
$h \geq r_1$



Formula:

$$F_{12} = \frac{1}{8} - \frac{\cos^{-1} Z}{2\pi\sqrt{1+R_2^2}} + \frac{1}{4\pi}\sin^{-1}\frac{1-R_2^2 Z^2 - 2Z^2}{1+R_2^2 Z^2} \qquad [4\text{-}67]$$

References: Feingold & Gupta (1970) [12], Siegel & Howell (1972) [37].

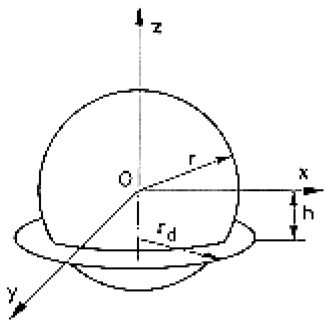


Note: non-si units are used in this figure

Figure 4-40: F_{12} as a function of Z for different values of R_2. Calculated by the compiler.

4.3.6.2.1 Sphere to intersecting coaxial disc

Sphere to both sides of an intersecting coaxial disc. The inner circle of the annular disc seats on the sphere.



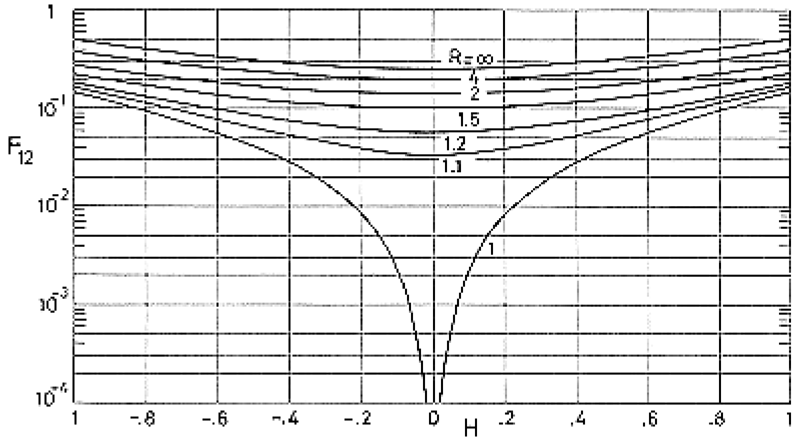
$H = h/r$

$R = r_d/r$

Formula:

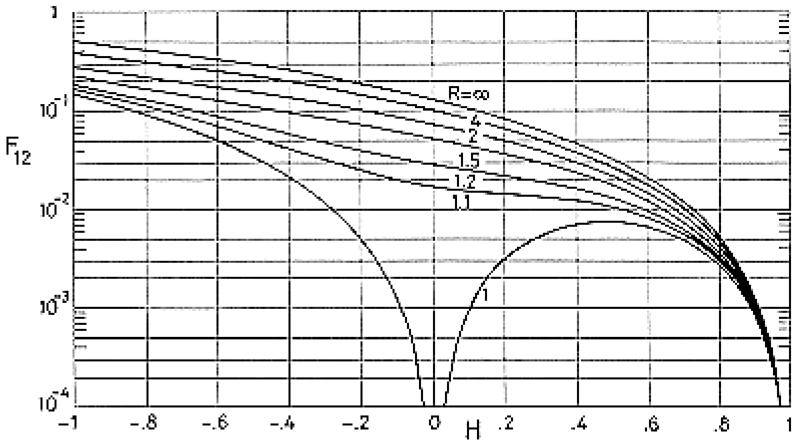
$$F_{12} = \frac{1}{2\pi}\left[\begin{array}{l} -\frac{\sqrt{(1-H^2)(R^2+H^2-1)}}{2} + \frac{R^2}{2}\tan^{-1}\sqrt{\frac{1-H^2}{R^2+H^2-1}} - \\ -\frac{(1+2H-H^2)\pi}{4} + \frac{H}{\sqrt{R^2+H^2}}\cos^{-1}\frac{H\sqrt{R^2+H^2-1}}{R} + \\ +\tan^{-1}\sqrt{\frac{R^2+H^2-1}{1-H^2}} \end{array}\right] \qquad [4\text{-}68]$$

Reference: Chung & Naraghi (1981) [5].



Note: non-si units are used in this figure

Figure 4-41: F_{12} from a sphere to both sides of a coaxial intersecting disc, vs. *H*, for different values of *R*. Calculated by the compiler.



Note: non-si units are used in this figure

Figure 4-42: F12 from a sphere to the upper side of a coaxial intersecting disc, vs. *H* (-1 ≤*H*≤ 1), for different values of *R*. Calculated by the compiler.

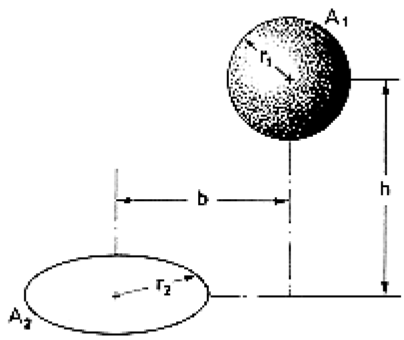
4.3.6.3 Sphere to non-coaxial disc

Sphere to a noncoaxial disc.

$R = r_2/h$

$Z = b/r_2$

$h \geq r_1$



Formula:

a. When $Z \geq 1$

$$F_{12} = \frac{1}{2\pi}\int_{Z-1}^{1+Z} \cos^{-1}\left(\frac{x^2 + Z^2 - 1}{2xZ}\right) xR^2\left(1 + R^2x^2\right)^{-3/2} dx \qquad [4\text{-}69]$$

b. When $Z < 1$

$$F_{12} = \frac{1}{2\pi}\int_{Z-1}^{1+Z} \cos^{-1}\left(\frac{x^2 + Z^2 - 1}{2xZ}\right) xR^2\left(1 + R^2x^2\right)^{-3/2} dx + \frac{1}{2}\left(1 - \frac{1}{\sqrt{1 + R^2(1-Z)^2}}\right) \qquad [4\text{-}70]$$

Reference: Feingold & Gupta (1970) [12].

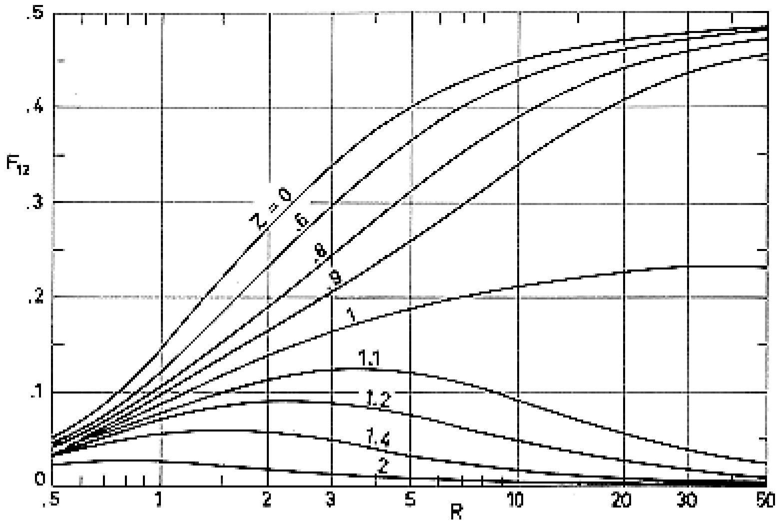
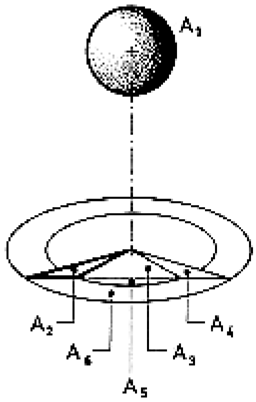


Figure 4-43: Values of F_{12} as a function of Z and R. Calculated by the compiler.

4.3.6.4 Sphere to arbitrary polygon

The following view factors may be obtained from those for a sphere viewing either a segment or a sector of a coaxial disc.

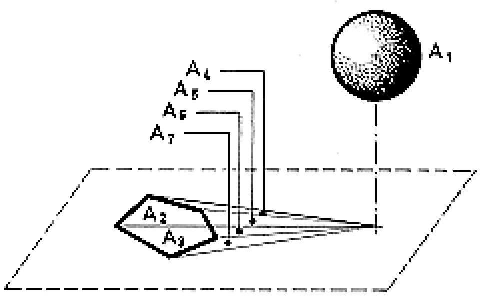


Sphere to general triangle with one vertex at the projection of the center of sphere on the plane of triangle. View factor is:

$F_{12} = [F_{1(2,3,4,5,6)} + F_{15} - F_{1(5,6)} - F_{1(3,5)}]/2$

Reference: Feingold & Gupta (1970) [12].

The view factor between a sphere and an arbitrary polygon may be obtained from the previous result.

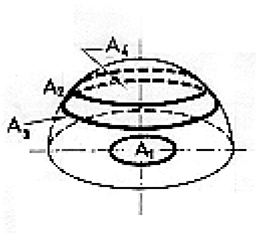


$F_{1(2,3)} = F_{1(2,4,5)} + F_{1(3,6,7)} - F_{14} - F_{15} - F_{16} - F_{17}$

Reference: Feingold & Gupta (1970) [12].

4.3.6.5 Axisymmetrical configurations

The following view factors may be obtained by use of those for two parallel circular discs.

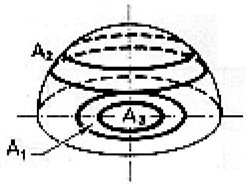


Disc on the base of a hemisphere to an axisymmetrical portion of the spherical surface.

$F_{12} = F_{13} - F_{14}$

Reference: Buschman & Pittman (1961) [3].

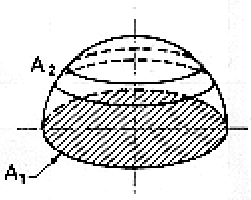
Ring on base of a hemisphere to an axisymmetrical portion of the spherical surface.



$$F_{12} = \left[1 + \frac{A_3}{A_1}\right] F_{(1,3)2} - \frac{A_3}{A_1} F_{32} \qquad [4\text{-}71]$$

Reference: Buschman & Pittman (1961) [3].

The following particular view factor can be obtained from the first one.



Axisymmetrical section of hemisphere to the base.

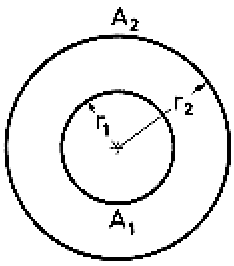
F_{12} =1/2.

Reference: Buschman & Pittman (1961) [3].

4.3.7 Cylindrical to cylindrical. two-dimensional configurations

4.3.7.1 Concentric circular cylinders

Two-dimensional concentric cylinders.



Formula:

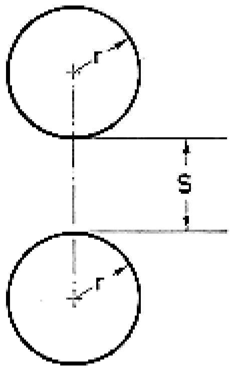
$F_{12} = 1$
$F_{21} = r_1/r_2$
$F_{22} = 1 - (r_1/r_2)$

References: Hamilton & Morgan (1952) [15], Siegel & Howell (1972) [37].

4.3.7.2 Parallel cylinders of the same diameter

Infinitely long parallel cylinders having the same diameter.

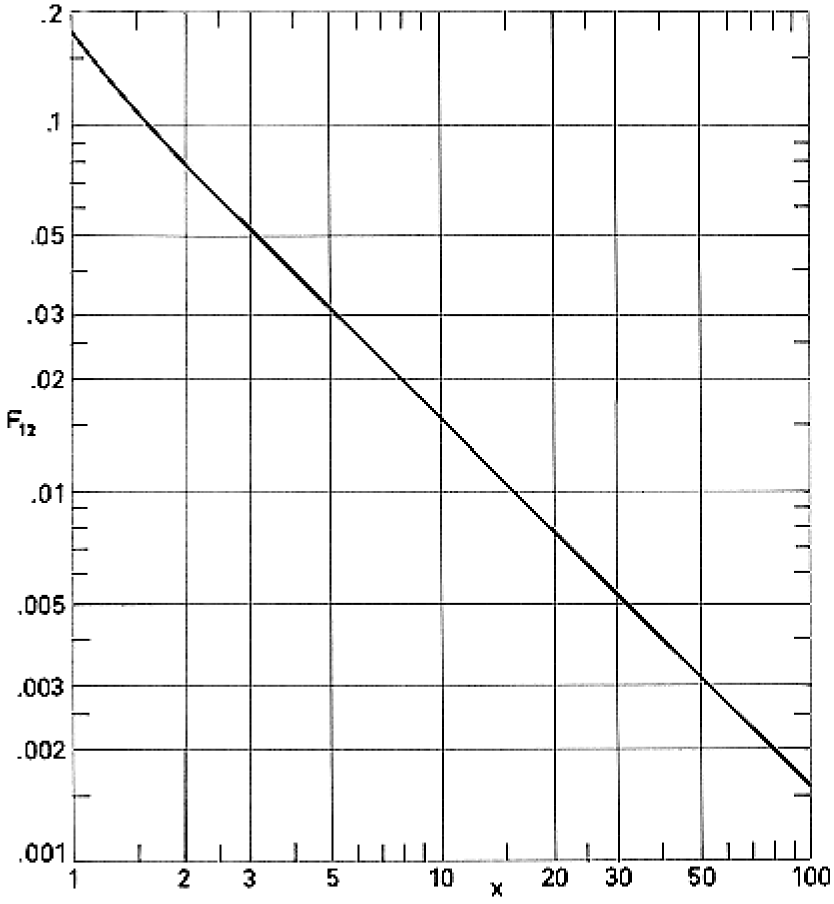
$x = 1 + S/2r$



Formula:

$$F_{12} = F_{21} = \frac{1}{\pi}\left[\sqrt{x^2 - 1} + \sin^{-1}\frac{1}{x} - x\right] \quad [4\text{-}72]$$

Reference: Siegel & Howell (1972) [37].



Note: non-si units are used in this figure

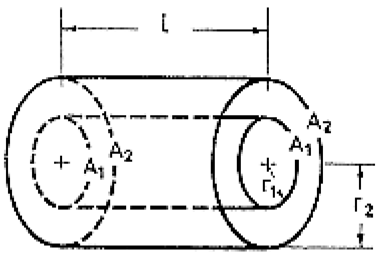
Figure 4-44: F12 as a function of x in the case of two infinitely long parallel cylinders of the same diameter. Calculated by the compiler.

4.3.8 Cylindrical to cylindrical. axisymmetrical configurations

4.3.8.1 Concentric circular cylinders of the same length

Two concentric cylinders of same finite length.

$R = r_2/r_1$
$L = l/r_1$



Formula:

$$F_{21} = \frac{1}{R} - \frac{1}{\pi R}\left\{\cos^{-1}\frac{B}{A} - \frac{1}{2L}\left[\begin{array}{l}\sqrt{(A+2)^2-(2R)^2}\,+\\ +\cos^{-1}\frac{B}{RA}+B\sin^{-1}\frac{1}{R}-\frac{\pi A}{2}\end{array}\right]\right\} \qquad [4\text{-}73]$$

$$\begin{aligned}F_{22} &= 1-\frac{1}{R}+\frac{2}{\pi R}\tan^{-1}\frac{2\sqrt{R^2-1}}{L}-\\ &-\frac{L}{2\pi R}\left[\begin{array}{l}\frac{\sqrt{4R^2+L^2}}{L}\sin^{-1}\frac{4(R^2-1)+(L^2/R^2)(R^2-2)}{L^2+4(R^2-1)}-\\ -\sin^{-1}\frac{R^2-2}{R^2}+\frac{\pi}{2}\left(\frac{\sqrt{4R^2+L^2}}{L}-1\right)\end{array}\right]\end{aligned} \qquad [4\text{-}74]$$

where for any argument ξ:

$-(\pi/2) \leq \sin^{-1}\xi \leq \pi/2$
$0 \leq \cos^{-1}\xi \leq \pi$

with

$A = L^2 + R^2 - 1$
$B = L^2 - R^2 + 1.$

$$\lim_{L\to\infty} F_{21} = \frac{1}{R} \qquad [4\text{-}75]$$

$$\lim_{L\to\infty} F_{22} = 1 - \frac{1}{R} \qquad [4\text{-}76]$$

References: Hamilton & Morgan (1952) [15], Leuenberger & Person (1956) [24], Siegel & Howell (1972) [37].

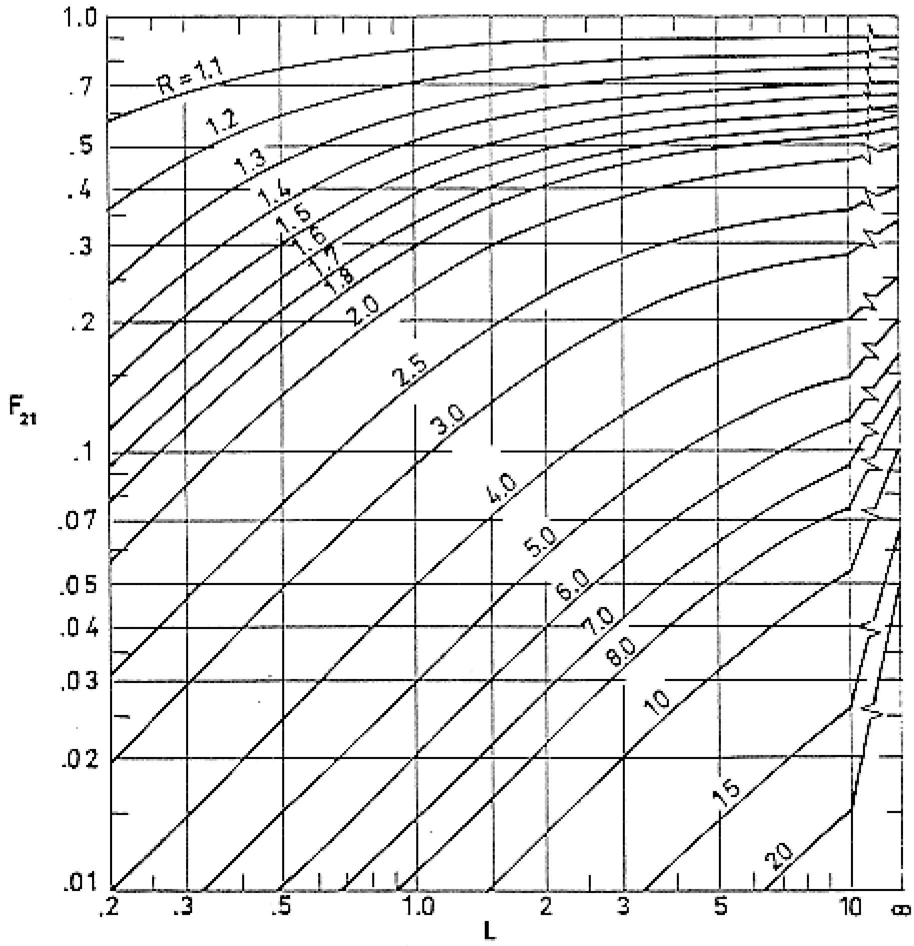


Figure 4-45: Plot of F_{12} vs. L for different values of *R*. From Hamilton & Morgan (1952) [15]

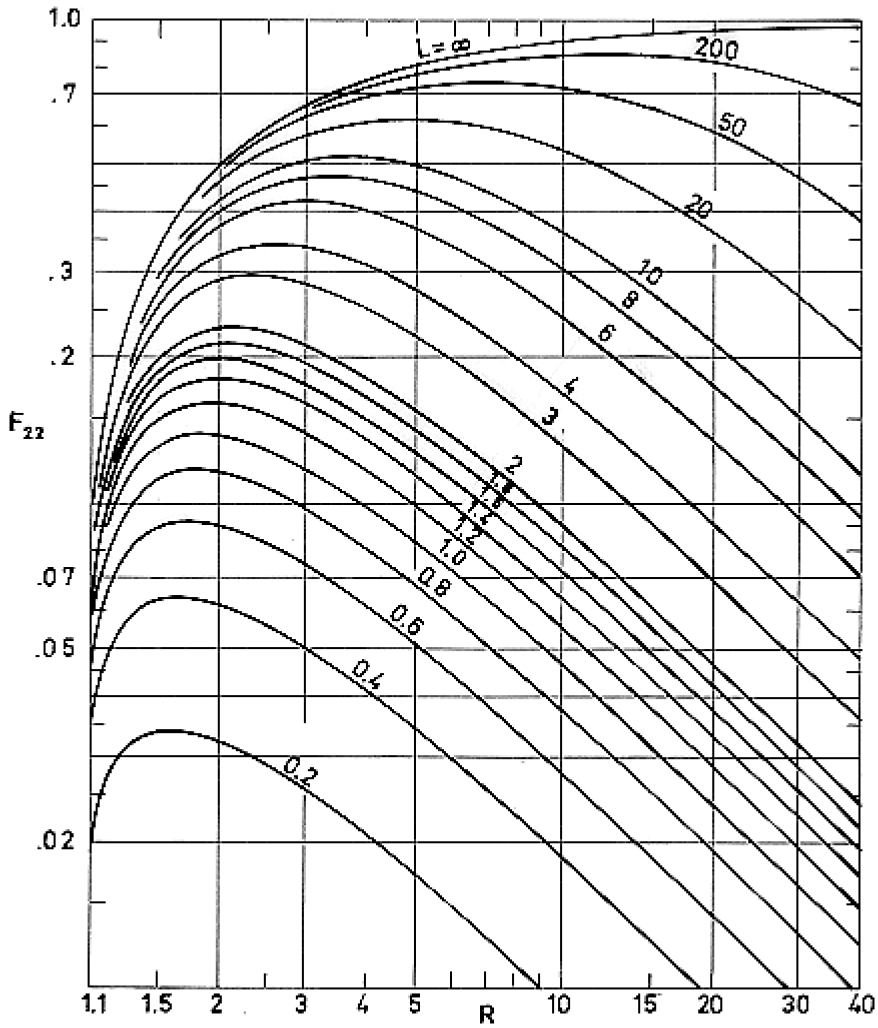
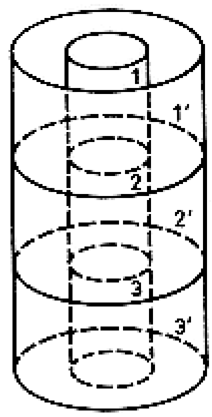


Figure 4-46: Plot of F_{22}, vs. R for different values of L. From Hamilton & Morgan (1952) [15]

4.3.8.2 Concentric circular cylinders of unequal length

The following view factors can be deduced by use of the results for two concentric cylinders of the same finite length.



Concentric cylinders of different finite length. The view factors are given by the following expressions:

$$A_2 F_{2(1',2',3')} = A_{2'} F_{2'(1,2,3)} = \frac{1}{2}\left[K_{(1,2)^2} + K_{(2,3)^2} - K_{1^2} - K_{3^2}\right] \qquad [4\text{-}77]$$

$$A_1 F_{13'} = A_3 F_{31'} = \frac{1}{2}\left[K_{(1,2,3)^2} - K_{(1,2)^2} - K_{(2,3)^2} - K_{2^2}\right] \qquad [4\text{-}78]$$

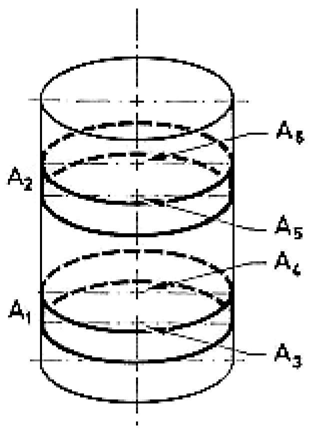
where

$k_{m2} = A_m F_{mm'}$.

Reference: Leuenberger & Person (1956) [24].

4.3.8.3 Finite areas in the same circular cylinder

The following view factor may be obtained from those for two parallel circular discs.

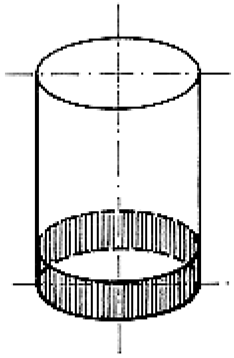


Finite ring area on interior of right circular cylinder to separate similar area

$$F_{12} = \frac{A_4}{A_1}\left[F_{45} - F_{46} - F_{35} + F_{36}\right] \qquad [4\text{-}79]$$

Reference: Buschman & Pittman (1961) [3].

The following view factor can be deduces as a particular case of the previous one.



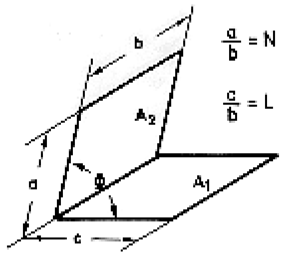
Portion of inner surface of cylinder to remainder of inner surface.

References: Leuenberger & Person (1956) [24], Buschman & Pittman (1961) [3].

4.3.9 Spherical to cylindrical

Sphere to the inner surface of a coaxial cylinder having equal or larger radius.

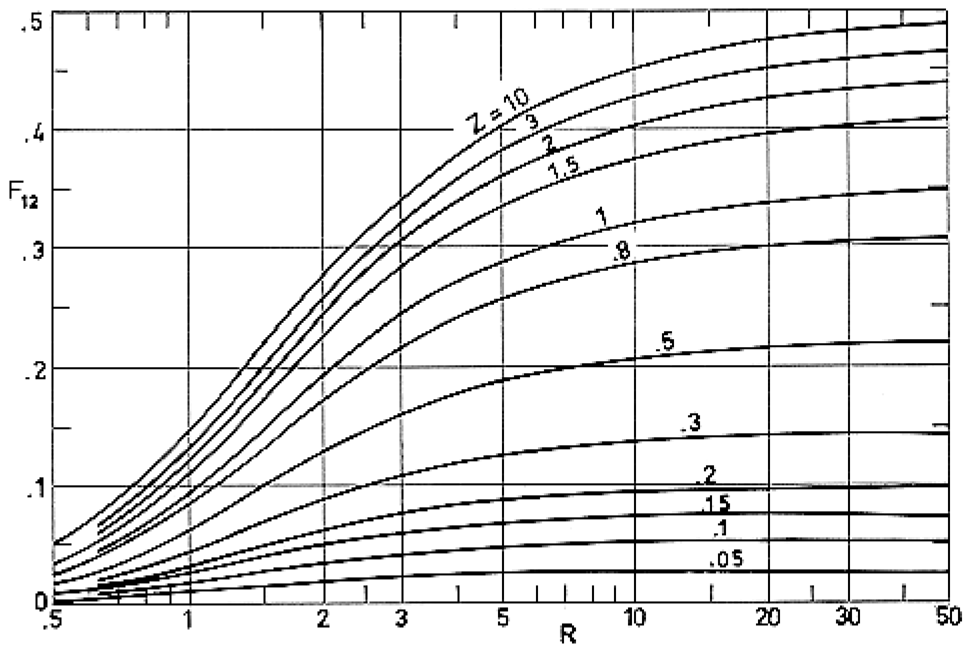
$R = r/a$
$Z = l/r$



Formula:

$$F_{12} = \frac{1}{2}\left[\frac{1+RZ}{\sqrt{(RZ+1)^2+R^2}} - \frac{1}{\sqrt{1+R^2}}\right] \qquad [4\text{-}80]$$

Reference: Feingold & Gupta (1970) [12].

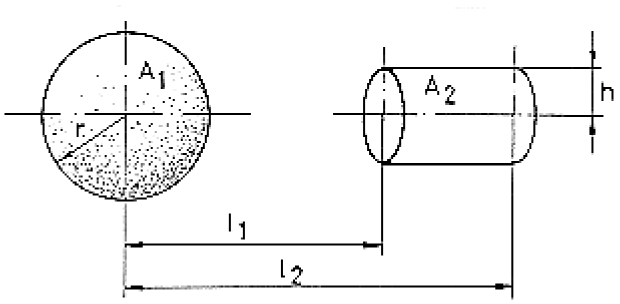


Note: non-si units are used in this figure

Figure 4-47: F_{12} as a function of R for different values of Z. Calculated by the compiler.

4.3.9.1 Sphere to external surface of cylinder

Sphere to external lateral surface of a coaxial cylinder:



$H = h/r$
$L_1 = l_1/r$
$L_2 = l_2/r$
$H \leq 1$
$L_2 > L_1 \geq 0$

Formula:

$$F_{12} = \frac{H}{2\pi}\left[f(L_2) - f(L_1)\right] \qquad [4\text{-}81]$$

where

$$f(L) = L\tan^{-1}\sqrt{\frac{1-H^2}{L^2+H^2-1}} - \\ -\frac{L}{H\sqrt{L^2+H^2}}\cos^{-1}\frac{H\sqrt{L^2+H^2-1}}{L} \quad [4\text{-}82]$$

Reference: Chung & Naraghi (1981) [5].

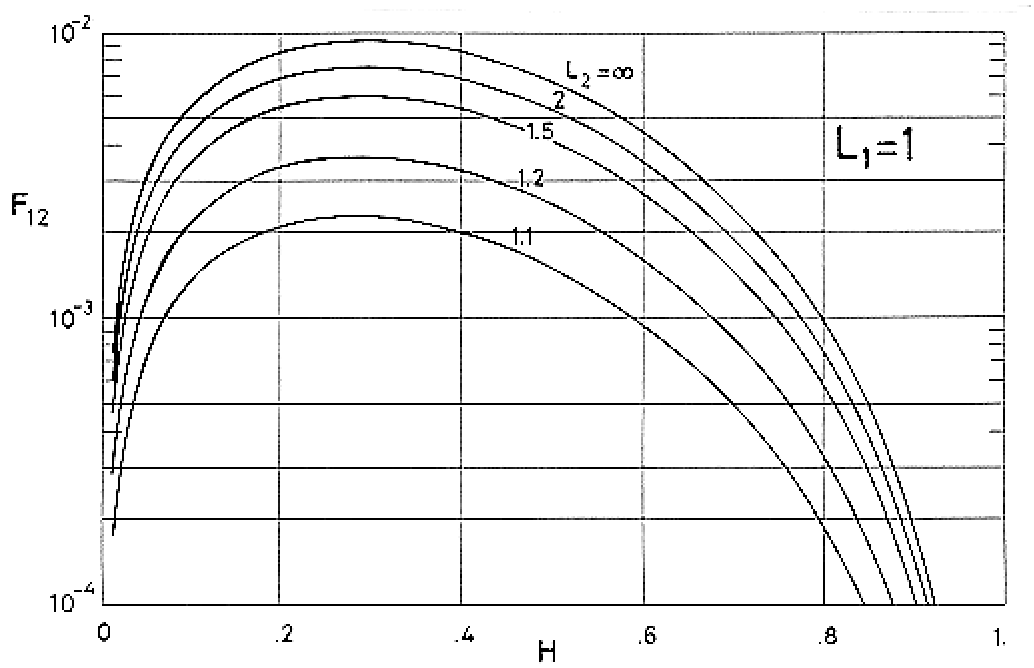
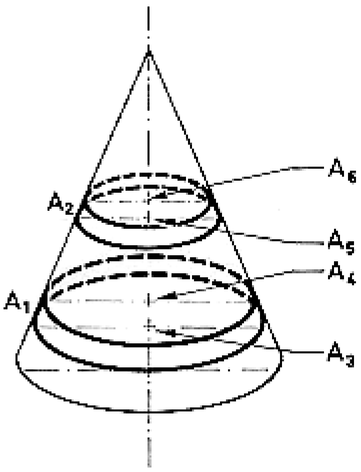


Figure 4-48: Values of F_{12} as a function of H and L_2 for $L_1 = 1$. Calculated by the compiler.

4.3.10 Conical to conical

The following view factor may be obtained from those corresponding to two parallel circular discs.



View factor between axisymmetrical sections of right circular cone

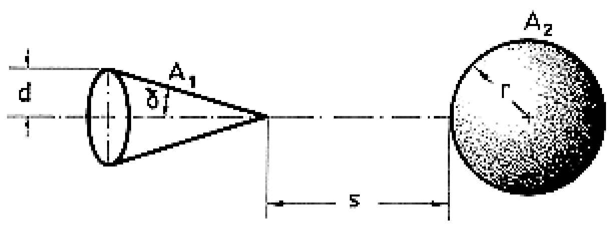
$$F_{12} = \frac{A_4}{A_1}\left(F_{45} - F_{46}\right) - \frac{A_3}{A_1}\left(F_{35} - F_{36}\right) \quad [4\text{-}83]$$

For F_{35}, F_{36}, F_{45} and F_{46} see clause 4.3.2.7.

Reference: Buschman & Pittman (1961) [3].

4.3.11 Conical to spherical

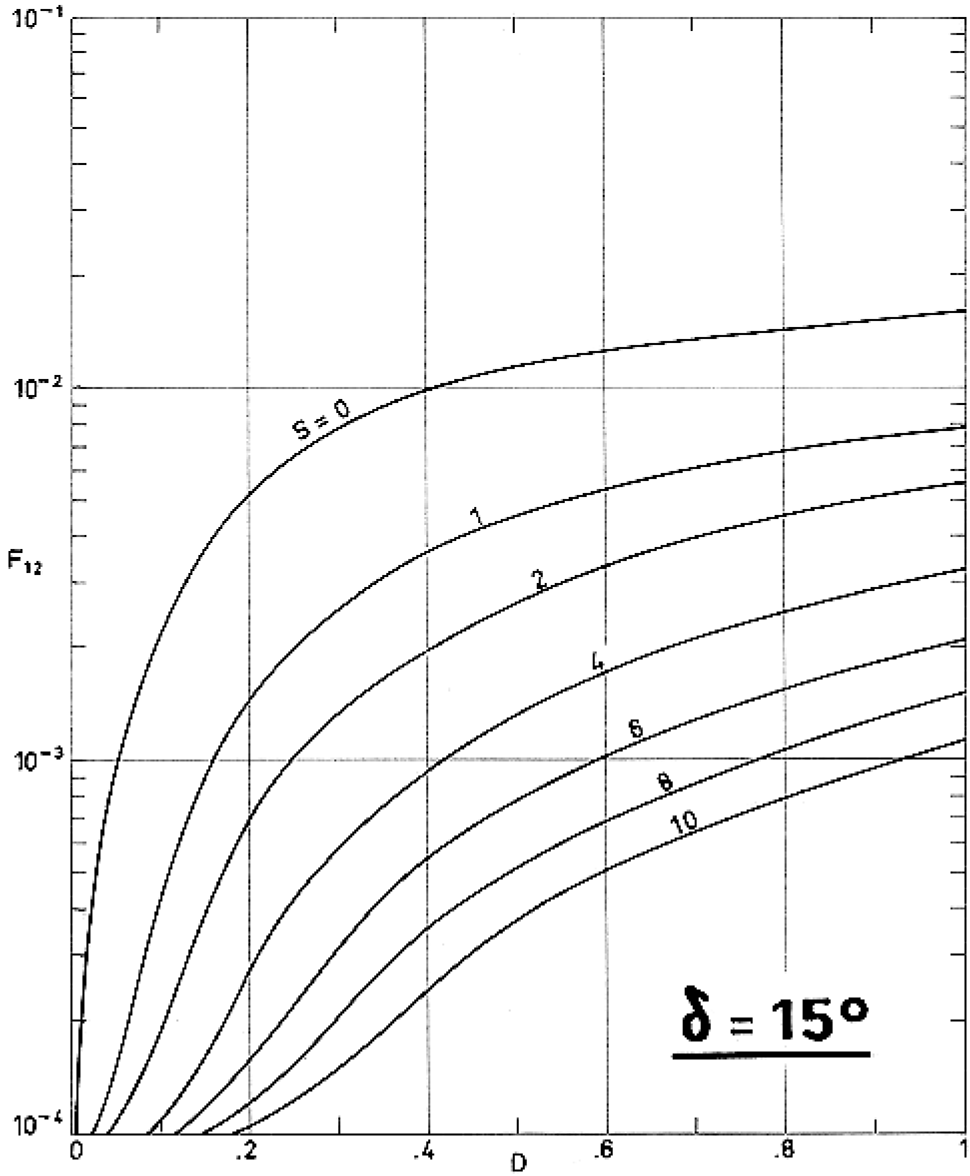
Cone to sphere; axis of cone passes through center of sphere.



$S = s/r$
$D = d/r$

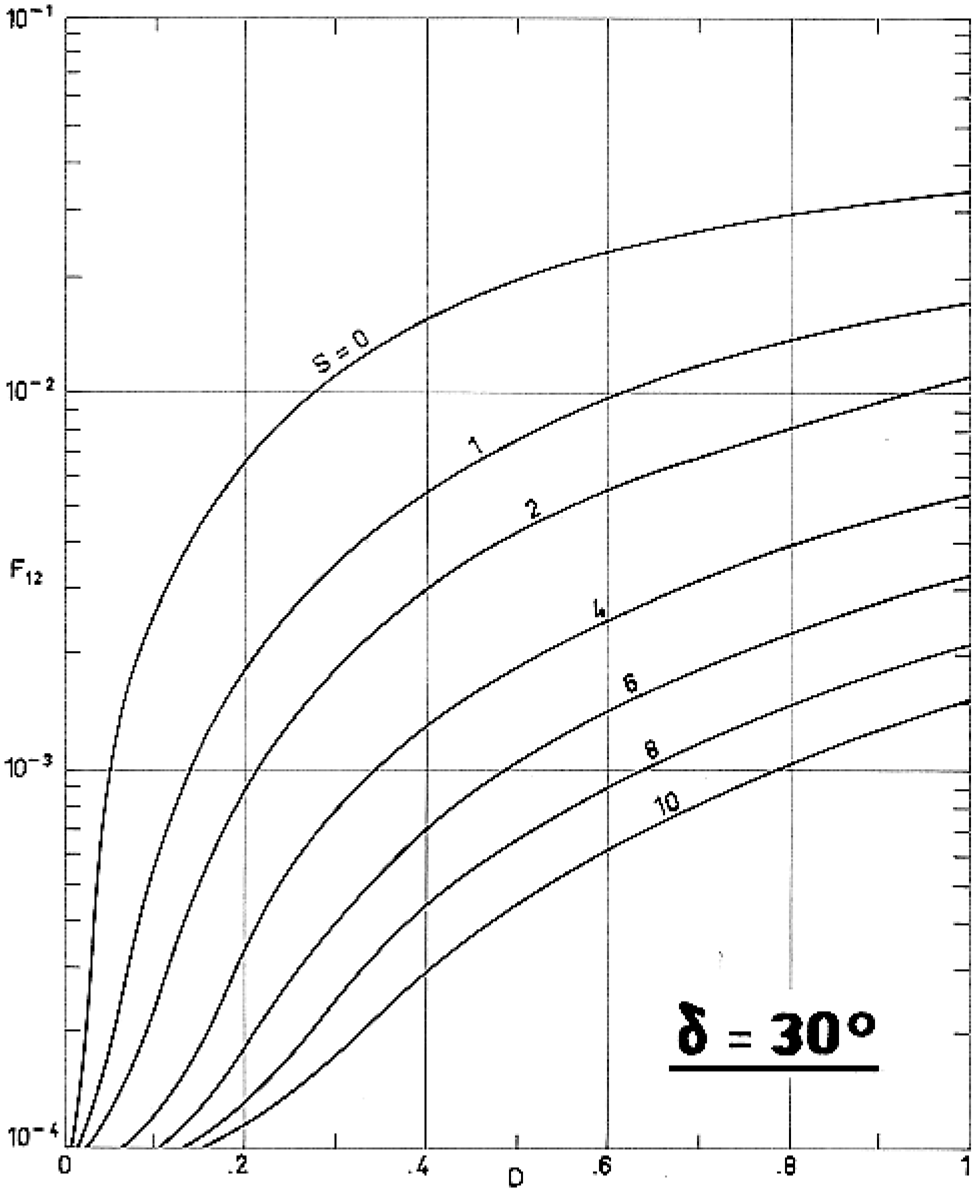
All results presented in the literature are obtained numerically.

Reference: Campbell & McConnell (1968) [4].



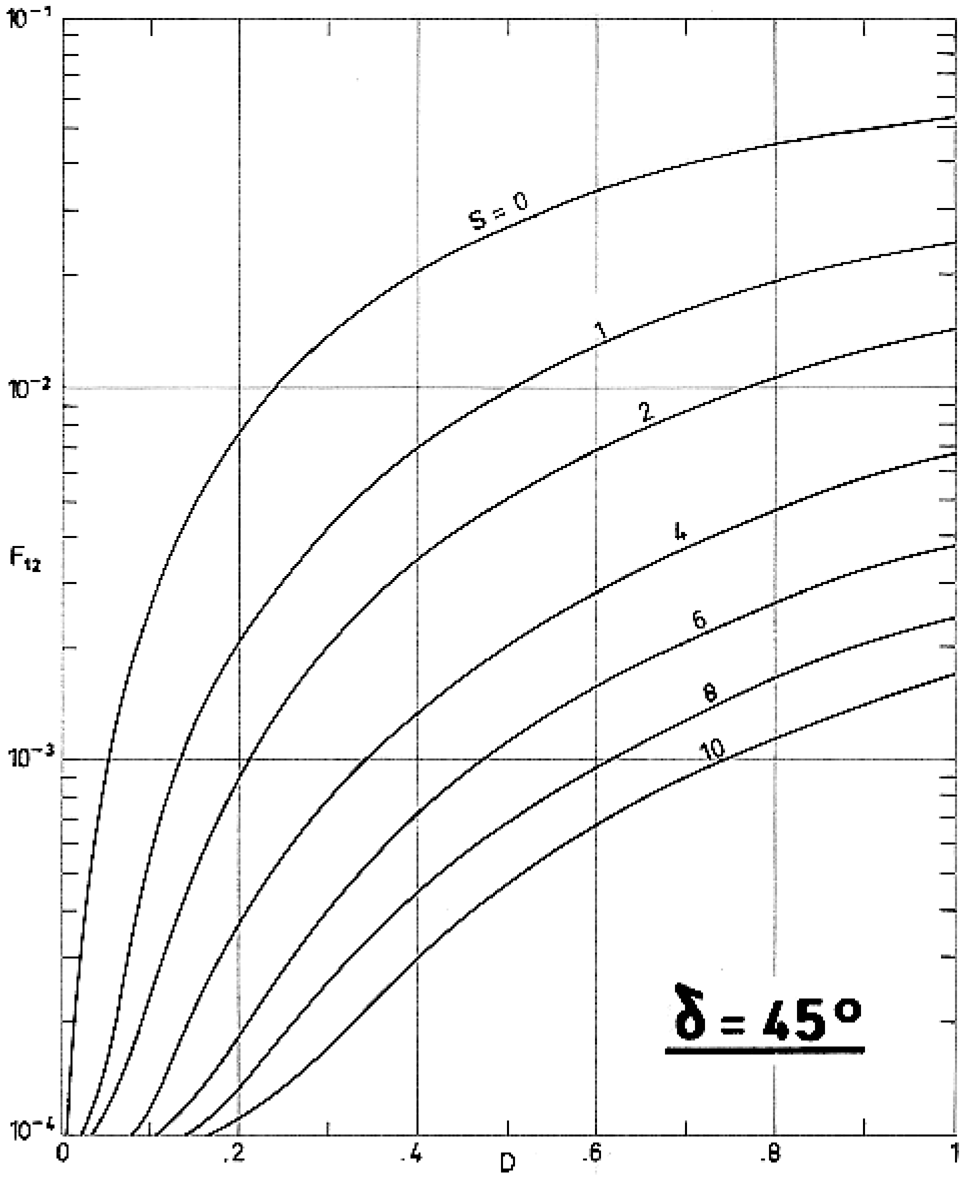
Note: non-si units are used in this figure

Figure 4-49: Values of F_{12} as a function of S and D, for δ = 15°. From Campbell & McConnell (1968) [4].



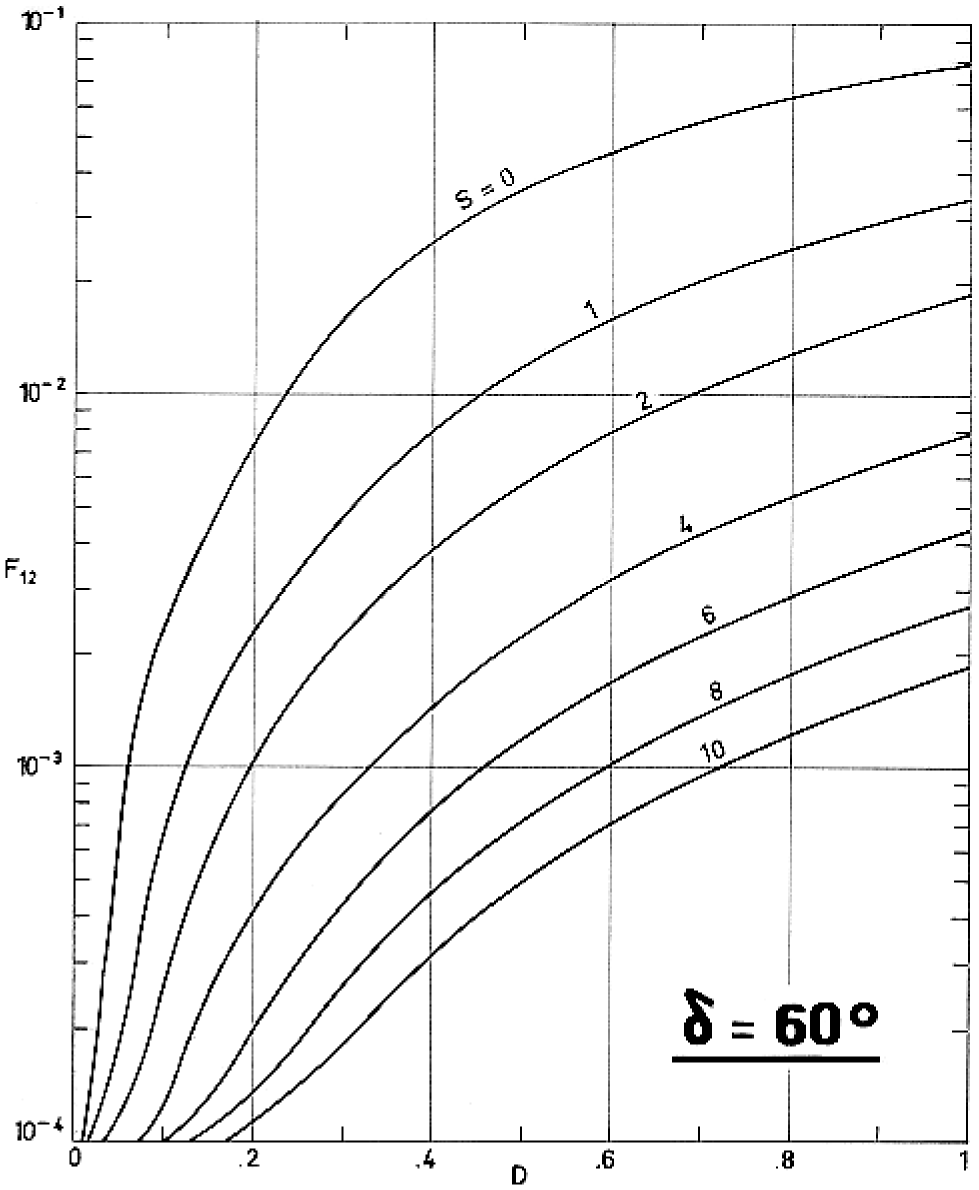
Note: non-si units are used in this figure

Figure 4-50: Values of F_{12} as a function of S and D, for δ = 30°. From Campbell & McConnell (1968) [4].



Note: non-si units are used in this figure

Figure 4-51: Values of F_{12} as a function of S and D, for δ = 45°. From Campbell & McConnell (1968) [4].



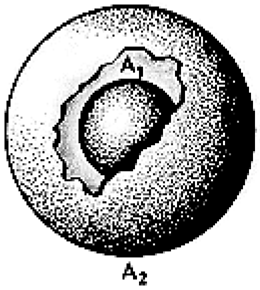
Note: non-si units are used in this figure

Figure 4-52: Values of F_{12} as a function of S and D, for $\delta = 60°$. From Campbell & McConnell (1968) [4].

4.3.12 Spherical to spherical

4.3.12.1 Concentric spheres

Concentric spheres; inner to outer sphere; outer to inner sphere; outer sphere to itself.



Formula:

$F_{12} = 1$

$F_{21} = (r_1/r_2)^2$

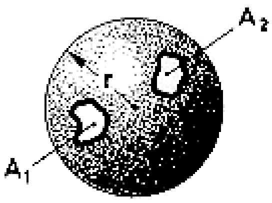
$F_{22} = 1- (r_1/r_2)^2$

where r_1 and r_2 are the radii of the spheres A_1 and A_2, respectively.

References: Hamilton & Morgan (1952) [15]; Kreith (1962) [22], Siegel & Howell (1972) [37].

4.3.12.2 Finite areas in the same spherical surface

Finite areas on interior of spherical cavity.

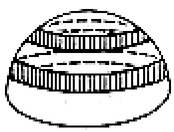


Formula:

$F_{12} = A_2/4\pi r_2$

References: Jakob (1957) [19], Siegel & Howell (1972) [37].

View factor between axisymmetrical sections of hemisphere can be deduced as a particular case of the previous one.

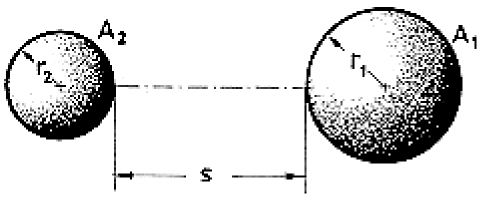


Reference: Buschman & Pittman (1961) [3].

4.3.12.3 Sphere to outer sphere

Sphere to sphere.

$S = s/r_1$
$R = r_2/r_1$



All results presented in the literature are obtained numerically.

References: Jones (1965) [21], Campbell & McConnell (1968) [4].

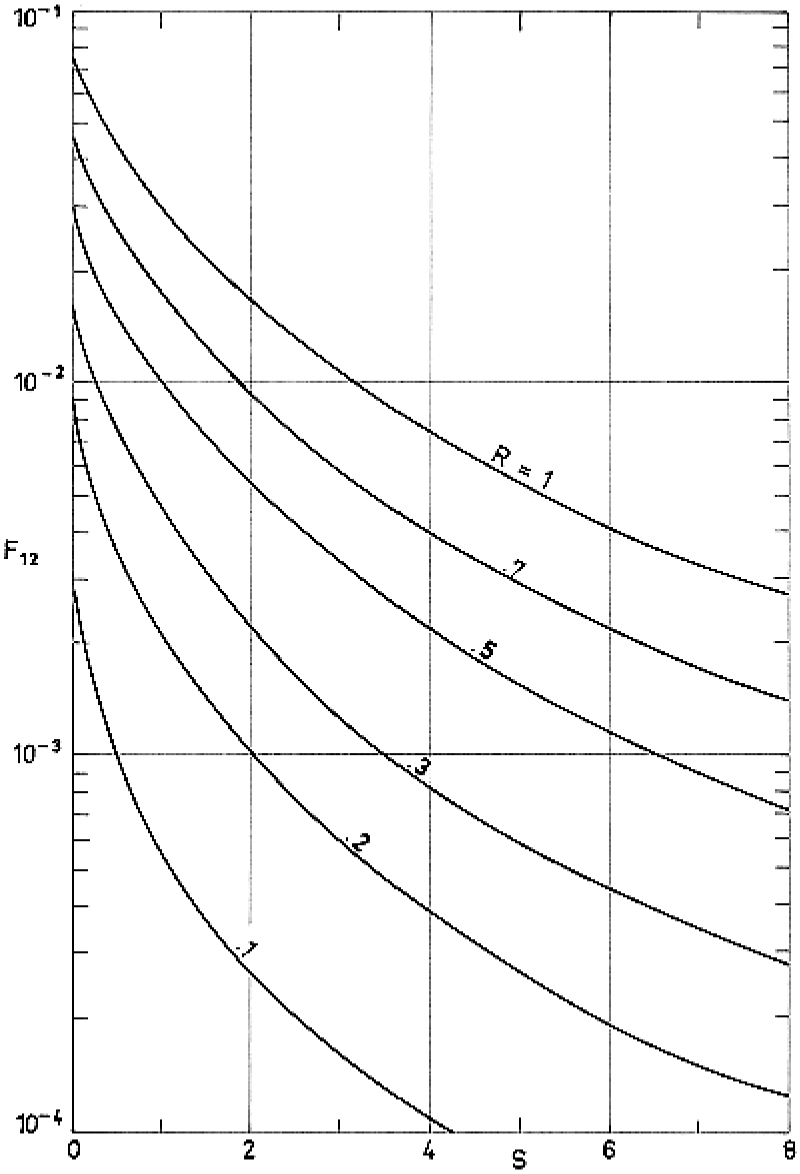
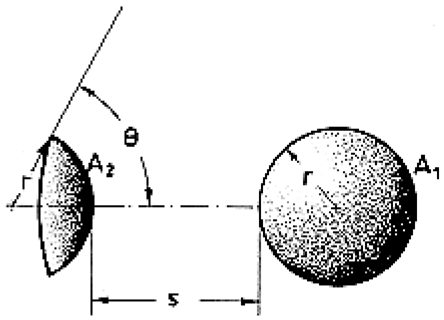


Figure 4-53: Values of F_{12} as a function of *S* and *R*. From Jones (1965) [21].

4.3.12.4 Sphere to cap on another sphere of equal radius

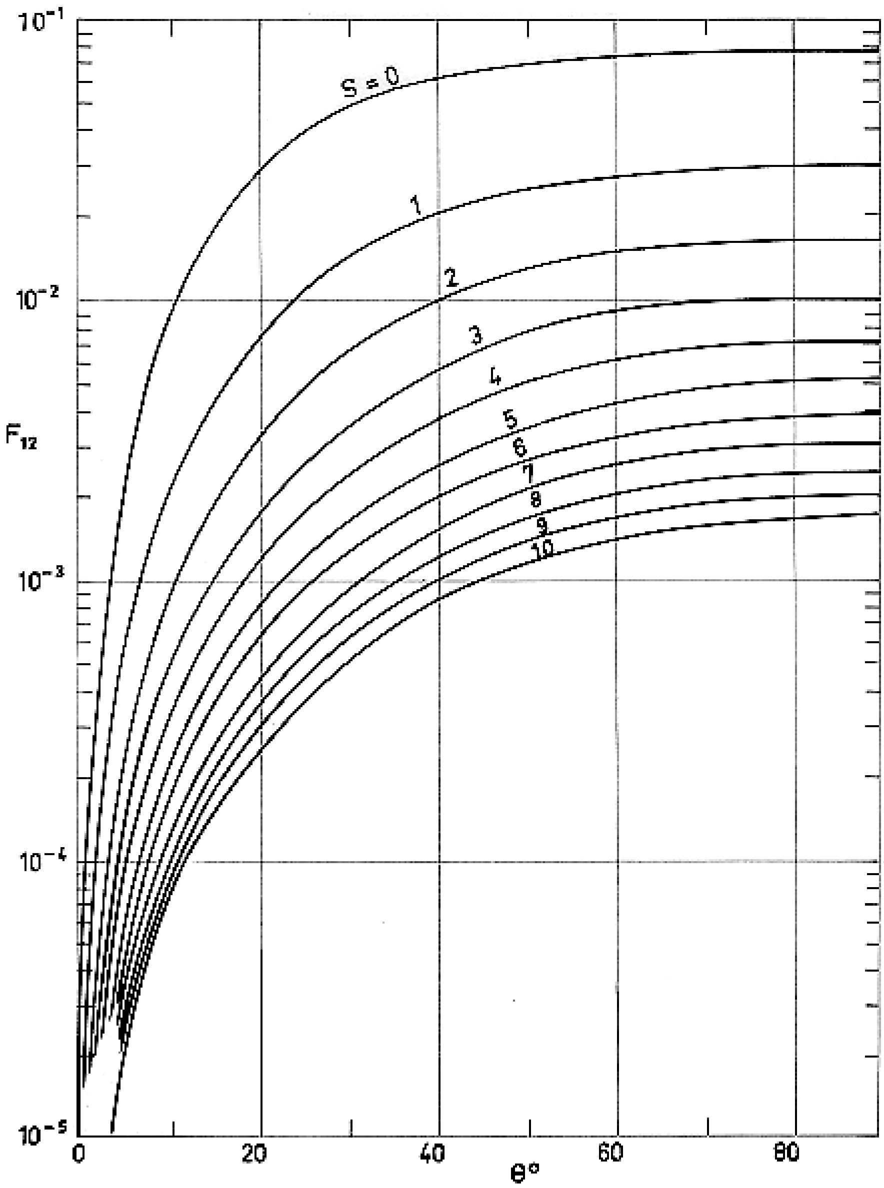
Sphere to cap on another sphere having equal radius and placed in an axisymmetrical fashion.

S = *s*/*r*



All results presented in the literature are obtained numerically.

References: Campbell & McConnell (1968) [4].



Note: non-si units are used in this figure

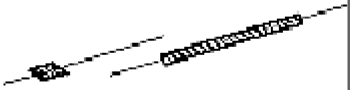
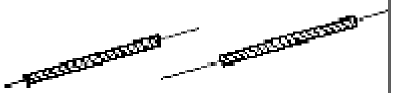
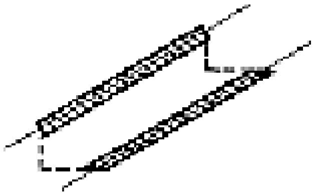
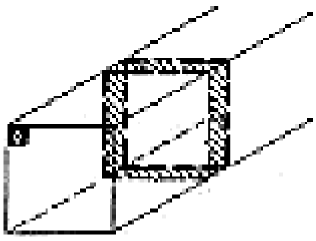
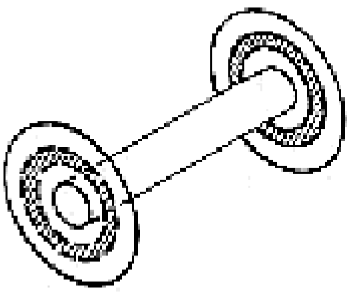
Figure 4-54: Values of F_{12} as a function of S and θ. From Campbell & McConnell (1968) [4].

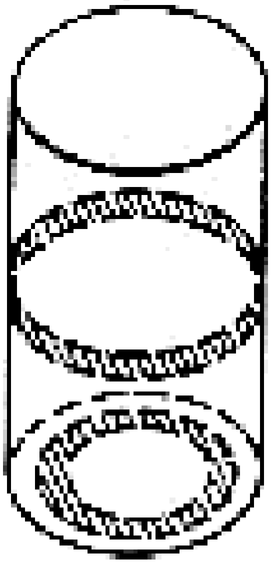
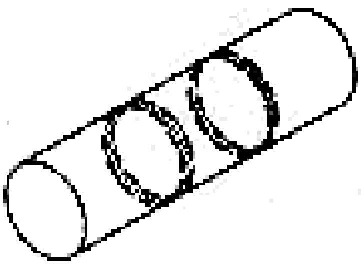
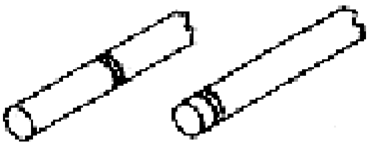
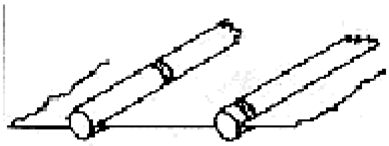
4.4 Additional sources of data

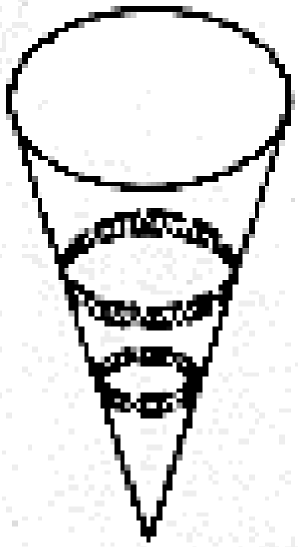
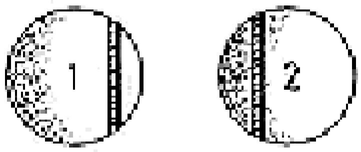
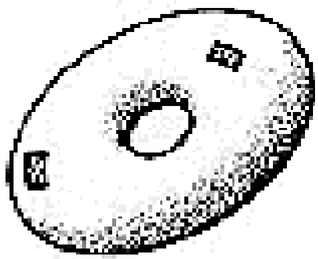
This clause contains information on view factors of several diffuse surface configurations not explicitly included in the previous data sheets.

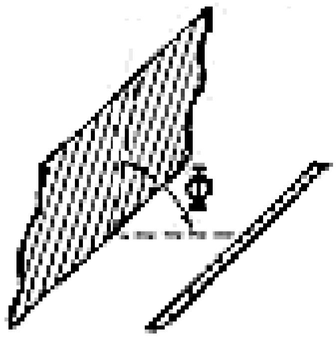
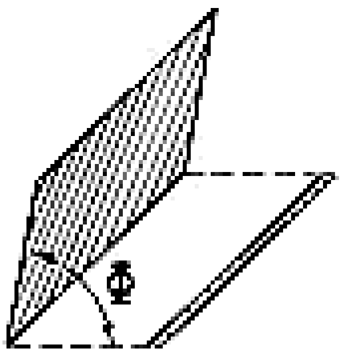
The reader is referred to the previously listed references whenever possible, only when a new source is required the reference is given with some detail the first time it appears on each page.

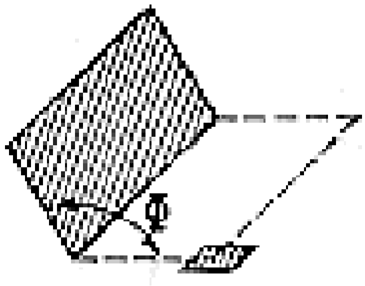
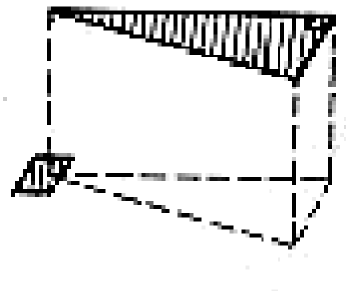
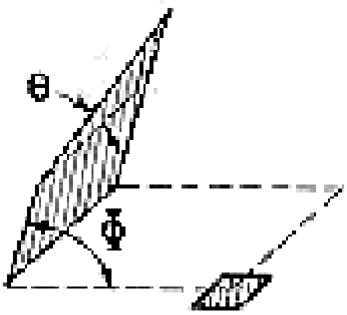
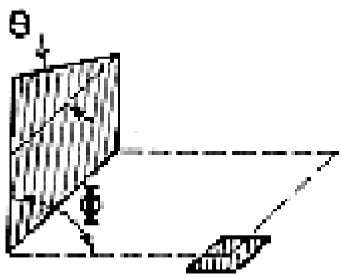
The tables presented below have been borrowed from Siegel & Howell (1972) [37]. No attempt has been made to include information which has been published subsequently.

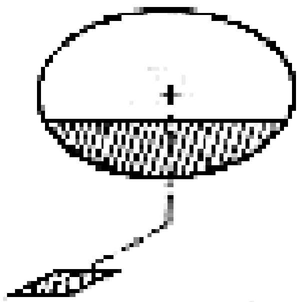
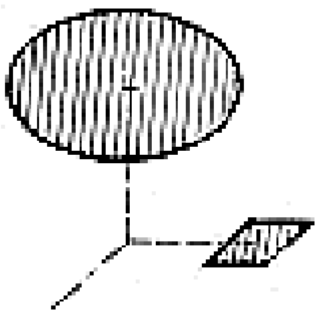
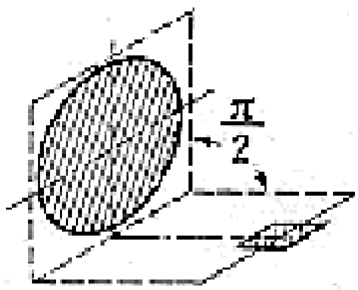
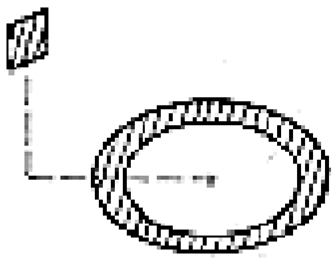
Definition	Sketch	Sources
Elemental area to infinitely long strip of differential width lying on parallel generating line.		Jakob (1957) [19], Siegel & Howell (1972) [37].
Infinitely long strip of differential width to similar strip on parallel generating line.		Jakob (1957) [19], Siegel & Howell (1972) [37].
Strip of finite and differential width to strip of same length on parallel generating line.		Siegel & Howell (1972) [37].
Corner element of end of square channel to sectional wall element on channel.		Siegel & Howell (1972) [37].
Ring element on fin to ring element on adjacent fin.		Sparrow, Miller & Jonsson (1962) [43].

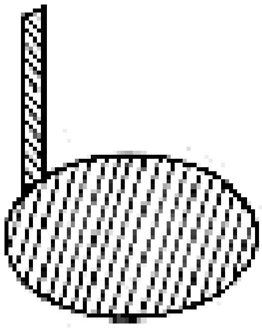
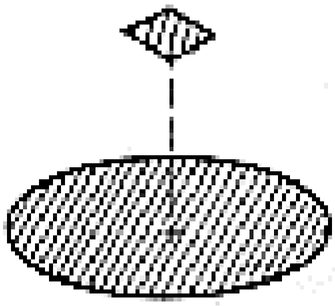
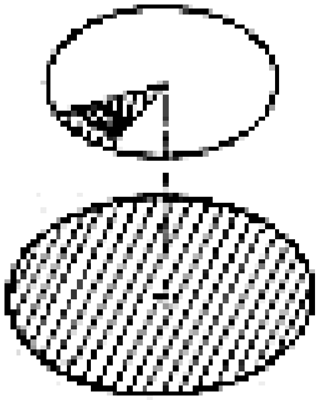
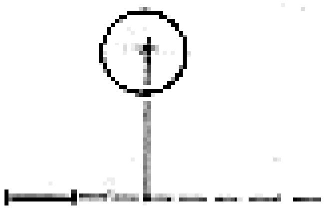
Definition	Sketch	Sources
Band of differential length on inside of cylinder to differential ring on cylinder base.		Sparrow, Albers & Eckert (1962) [40]. Journal of Heat Transfer vol. 84c, No. 1, 1962, pp. 73-81.
Two ring elements on interior of right circular cylinder.		Siegel & Howell (1972) [37].
Exterior element on tube surface to exterior element on adjacent parallel tube of same diameter.		Sparrow & Jonsson (1963a) [44]. Journal of Heat Transfer vol. 85, No. 4, 1963, pp. 382-384.
Exterior element on partitioned tube to similar element on adjacent parallel tube of same diameter.		Sparrow & Jonsson (1963a) [44].

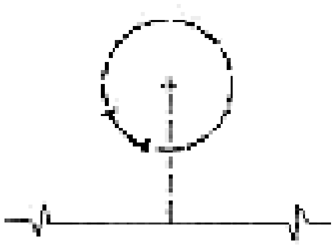
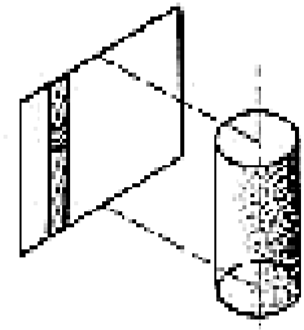
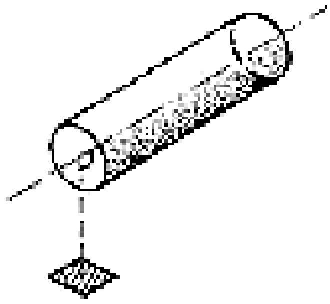
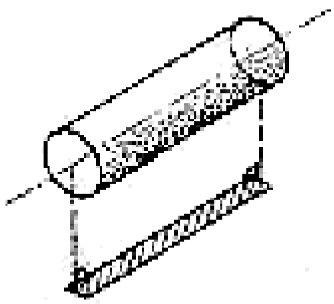
Definition	Sketch	Sources
Two elements on interior of right circular cone.		Sparrow & Jonsson (1963b) [45]. Journal of the Optical Society of America, vol. 53, No. 7, 1963, pp. 816-821.
Band on outside of sphere to band on another sphere.		($R_1 = R_2$), Campbell & McConnell (1968) [4]; ($R_1{}^1R_2$), Grier (1969) [13]. NASA SP-3050, 1969 [28].
Two differential elements on exterior of toroid.		Grier & Sommers (1969) [14]. NASA TN D-5006, 1969 [32].
Element on exterior of toroid to ring element on exterior of toroid.		Grier & Sommers (1969) [14].

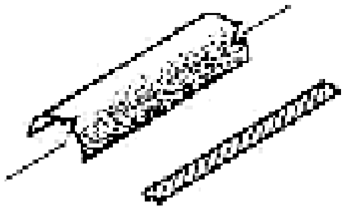
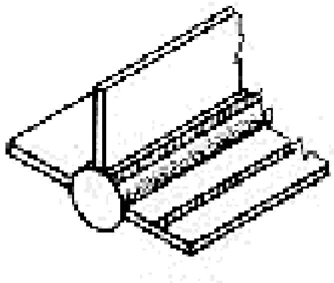
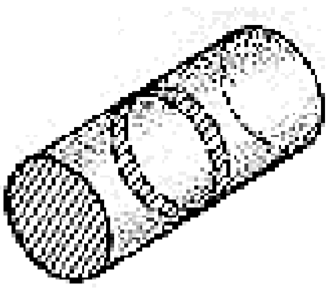
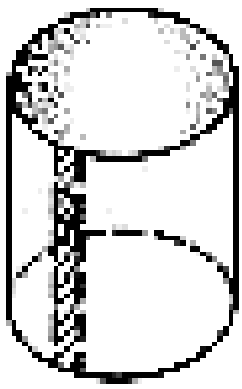
Definition	Sketch	Sources
Element on exterior of toroid to hoop element on exterior of toroid.		Grier & Sommers (1969) [14].
Plane strip element of any length to plane of finite width and infinite length.		Siegel & Howell (1972) [37].
Strip element of finite length to plane rectangle that intercepts plane of strip at angle *F* and width one edge parallel to strip.		Hamilton & Morgan (1952) [15], Kreith (1962) [22].
Area element to any parallel rectangle.		Jakob (1957) [19].

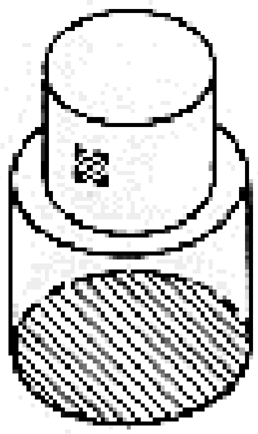
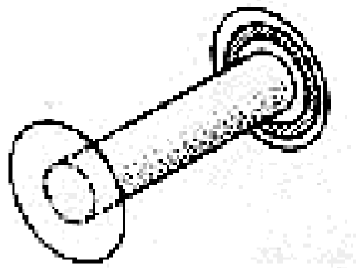
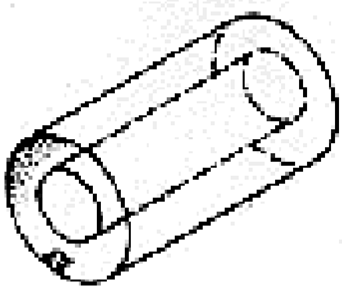
Definition	Sketch	Sources
Plane element to plane rectangle; planes containing two surfaces intersect at angle *F*.		Hamilton & Morgan (1952) [15], Kreith (1962) [22].
Plane element to right triangle in plane parallel to plane of element; normal to element passes through vertex of triangle.		Siegel & Howell (1972) [37].
Plane element to plane area with added triangular area; element is on corner of rectangle with one side in common with plane at angle *F*.		Hamilton & Morgan (1952) [16], Kreith (1962) [22].
Same geometry as preceding with triangle reversed relative to plane element.		Hamilton & Morgan (1952) [15], Kreith (1962) [22].

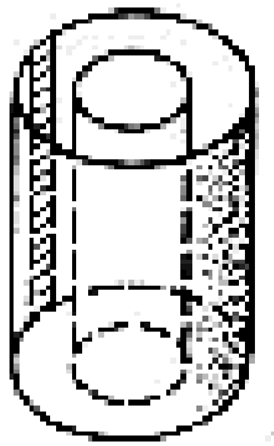
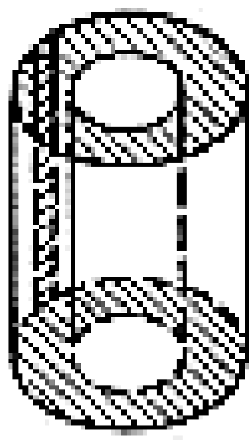
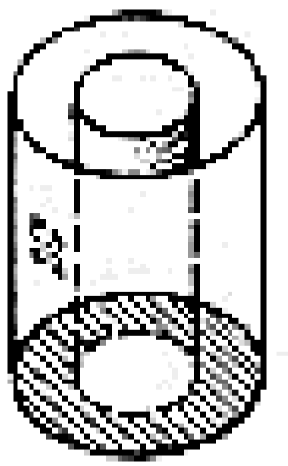
Definition	Sketch	Sources
Plane element to segment of disc in plane parallel to element.		Sparrow & Cess (1966) [41]. "Radiation Heat Transfer", Brooks/Cole Publishing Company, Belmont, California, 1966.
Plane element to circular disc on plane parallel to that of element.		Hamilton & Morgan (1952) [15], Jakob (1957) [19], Kreith (1962) [22], Siegel & Howell (1972) [37].
Plane element to circular disc; planes containing element and disc intersect at 90°, and centers of element and disc lie in plane perpendicular to those containing areas.		Hamilton & Morgan (1952) [15], Leuenberger & Person (1956) [24], Kreith (1962) [22], Siegel & Howell (1972) [37].
Plane element to ring area in plane perpendicular to element.		Siegel & Howell (1972) [37].

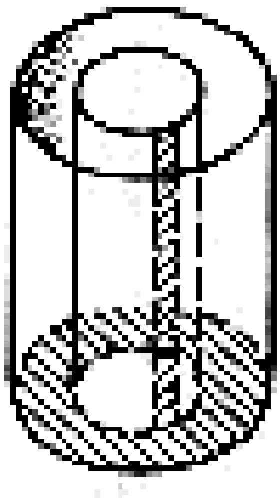
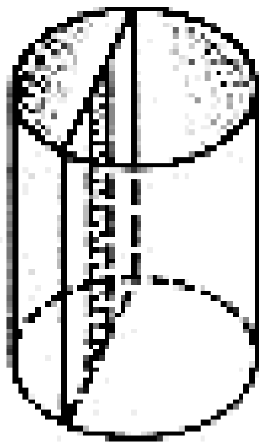
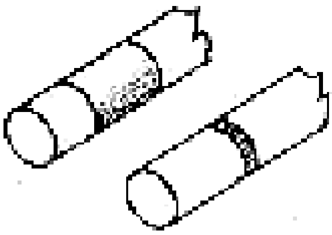
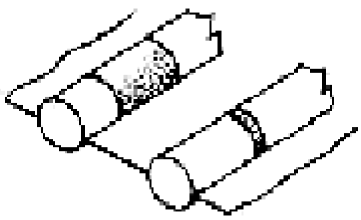
Definition	Sketch	Sources
Strip element of finite length to perpendicular circular disc located at one end of strip.		Leuenberger & Person (1956) [24].
Area element to parallel elliptical plate.		Moon (1961) [26], Siegel & Howell (1972) [37].
Radial and wedge elements on circular disc to disc in parallel plane.		Leuenberger & Person (1956) [24].
Infinite cylinder to parallel infinitely long strip element.		Feingold & Gupta (1970) [12], Siegel & Howell (1972) [37].

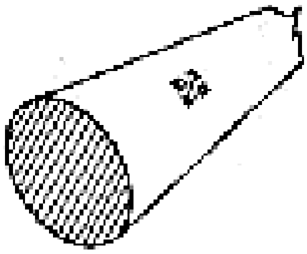
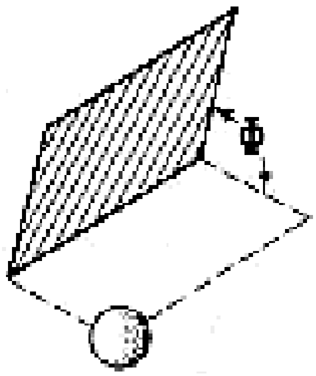
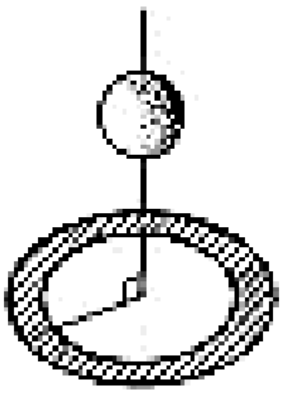
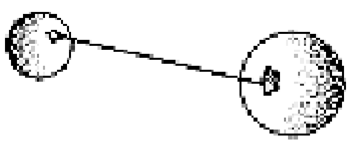
Definition	Sketch	Sources
Plane of infinite width and infinite length to infinitely long strip on the surface of a parallel cylinder.		Hamilton & Morgan (1952) [15], Kreith (1962) [22], Siegel & Howell (1972) [37].
Strip or element on plane parallel to cylinder axis to cylinder of finite length.		Leuenberger & Person (1956) [24].
Plane element to right circular cylinder of finite length; normal to element passes through center of one end of cylinder and is perpendicular to cylinder axis.		Hamilton & Morgan (1952) [15], Kreith (1962) [22], Siegel & Howell (1972) [37].
Elemental strip of finite length to parallel cylinder of same length; normals at ends of strip pass through cylinder axis.		Hamilton & Morgan (1952) [15], Leuenberger & Person (1956) [24], Kreith (1962) [22].

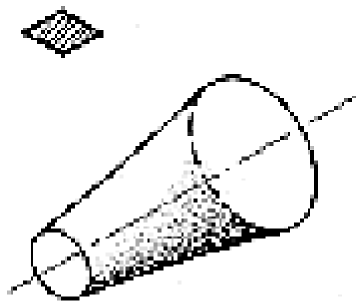
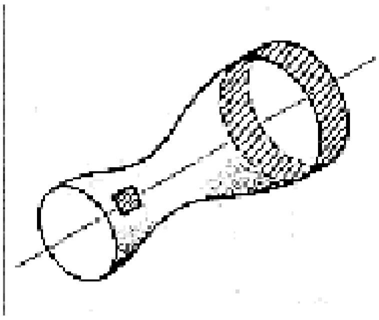
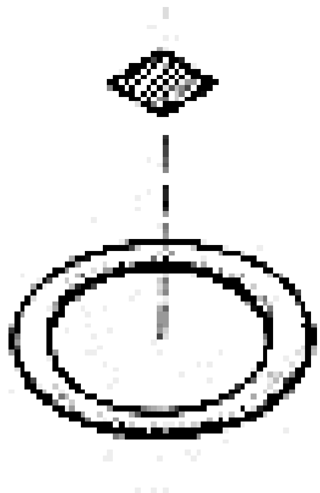
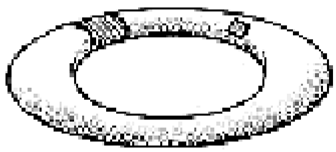
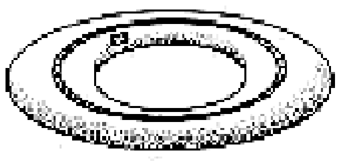
Definition	Sketch	Sources
Infinitely long strip of differential width to parallel semicylinder.		Sparrow & Eckert (1962) [42]. Journal of Heat Transfer, vol. 84, No. 1, 1968, pp. 12-18.
Infinite strip on any side of any of three fins to tube or environment, and infinite strip on tube to fin or environment.		Holcomb & Lynch (1967) [16]. Report ORNL-TM-1613, Oak Ridge National Laboratory, 1967.
Ring element on interior of right circular cylinder to end of cylinder.		Siegel & Howell (1972) [37].
Element and strip element on interior of finite cylinder to interior of cylindrical surface.		Leuenberger & Person (1956) [24].

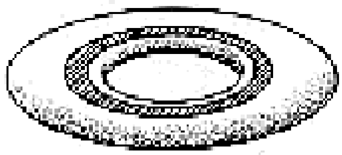
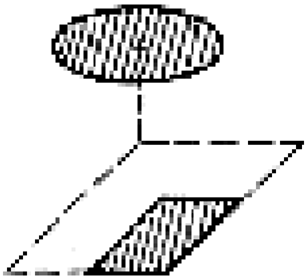
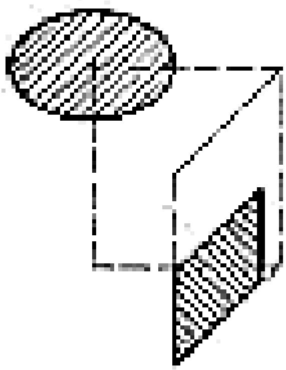
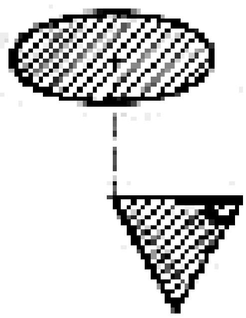
Definition	Sketch	Sources
Area element on interior of cylinder to base of second concentric cylinder; cylinders are one atop other.		Leuenberger & Person (1956) [24].
Ring element on fin to tube.		Sparrow, Miller & Jonsson (1962) [43].
Element is at end of wall on inside of finite length cylinder enclosing concentric cylinder of same length; factor is from element to inside surface of outer cylinder.		Hamilton & Morgan (1952) [15], Leuenberger & Person (1956) [24], Kreith (1962) [22].

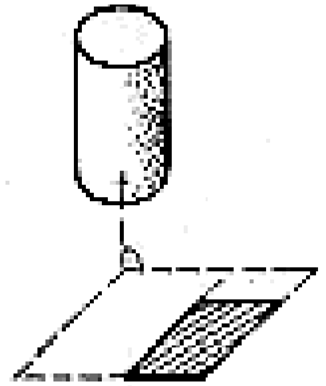
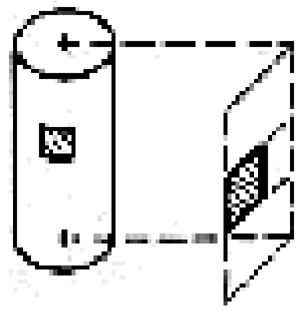
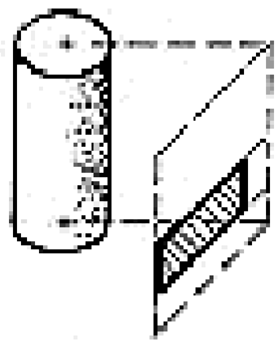
Definition	Sketch	Sources
Elemental strip on inner surface of outer concentric cylinder to interior surface of outer concentric cylinder.		Hamilton & Morgan (1952) [15], Leuenberger & Person (1956) [24], Kreith (1962) [22].
Elemental strip on inner surface of outer concentric cylinder to either annular end.		Hamilton & Morgan (1952) [15], Leuenberger & Person (1956) [24], Kreith (1962) [22].
Element on inside of outer finite concentric cylinder to inside cylinder or annular end.		Leuenberger & Person (1956) [24].

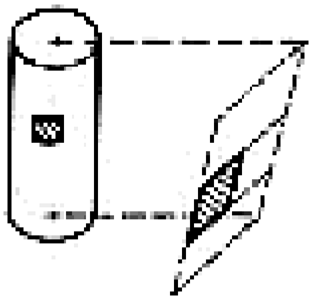
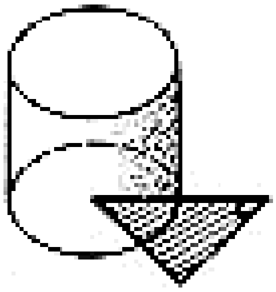
Definition	Sketch	Sources
Strip element on exterior of inner finite length concentric cylinder to inside of outer cylinder or to annular end.		Leuenberger & Person (1956) [24].
Strip on plane inside cylinder of finite length to inside of cylinder.		Leuenberger & Person (1956) [24].
Exterior element on tube surface to finite area on adjacent parallel tube of same diameter.		Sparrow & Jonsson (1963a) [44]. Journal of Heat Transfer, vol. 85, No. 4, 1963, pp. 382-384.
Exterior element on tube surface of partitioned tube to finite area on adjacent parallel tube of same diameter.		Sparrow & Jonsson (1963a) [44].

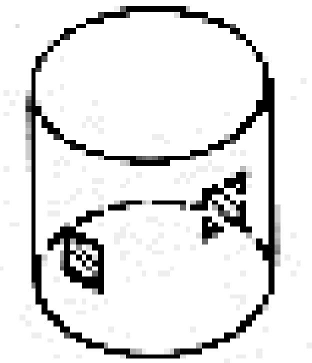
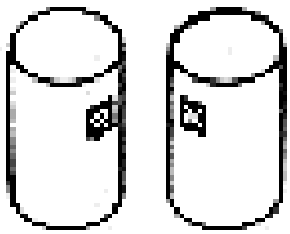
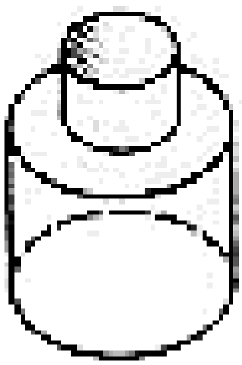
Definition	Sketch	Sources
Element on wall of right circular cone to base of cone.		Joerg & McFarland (1962) [20]. Report S62-245, Aerojet-General Corporation, 1962.
Spherical point source to rectangle. Point source is on one corner of rectangle that intersects with receiving rectangle at angle *F*.		Hamilton & Morgan (1952) [15], Jakob (1957) [19], Kreith (1962) [22].
Sphere to ring element oriented normal to sphere axis.		Feingold & Gupta (1970) [12].
Elemental area on sphere to finite area on second sphere.		Grier (1969) [13]. NASA SP-3050, 1969 [28].

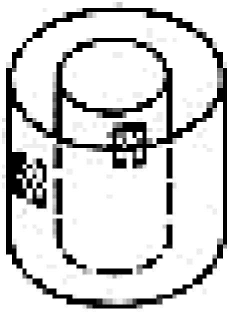
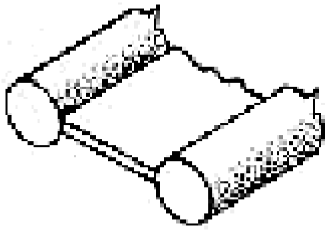
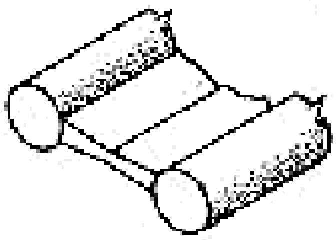
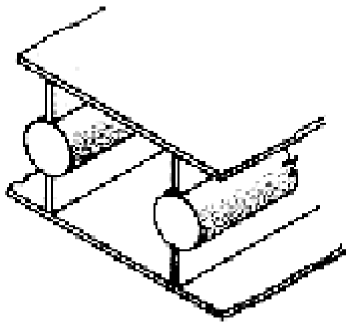
Definition	Sketch	Sources
Area element to axisymmetric surface-paraboloid, cone, cylinder (formulation given-factors are not evaluated).		Morizumi (1964) [27]. AIAA Journal, vol. 2, No. 11, 1964, pp. 2028-2030.
Element on interior (or exterior) of any axisymmetric body of revolution to band of finite length on interior (or exterior).		Robbins (1961) [35]. NASA TN D-586, 1961 [29]. Robbins & Todd (1962) [36]. NASA TN D-878, 1962 [30].
Slender torus to element on perpendicular axis.		Moon (1961) [26].
Element on exterior of toroid to toroidal segment of finite width.		Grier & Sommers (1969) [14]. NASA TN D-5006, 1969 [32].
Element on exterior of toroid to toroidal band of finite width.		Grier & Sommers (1969) [14].

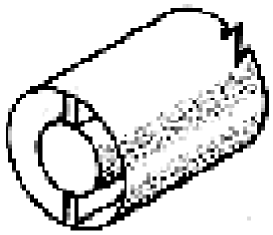
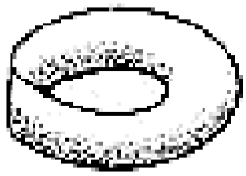
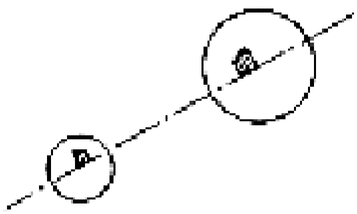
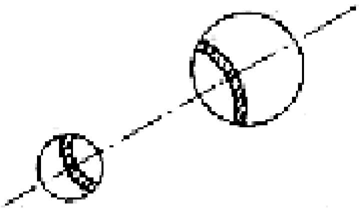
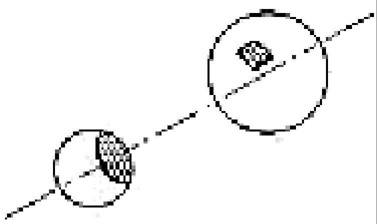
Definition	Sketch	Sources
Element and ring element on exterior of toroid to entire exterior of toroid.		Grier & Sommers (1969) [14]. Sommers & Grier (1969) [38]. Journal of Heat Transfer, vol. 91, No. 3, 1969, pp. 459-461.
Circular disc to arbitrarily placed rectangle in parallel plane.		Tripp, Hwang & Crank (1962) [48]. Special Report 16, Kansas State University Bulletin, vol. 46, No. 4, 1962.
Circle to arbitrarily placed rectangle in plane parallel to normal to circle.		Tripp, Hwang & Crank (1962) [48].
Circular disc to parallel right triangle; normal from center of circle passes through one acute vertex.		Tripp, Hwang & Crank (1962) [48].

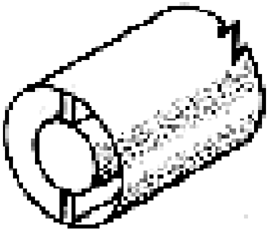
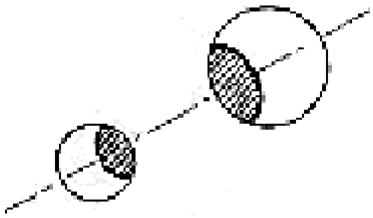
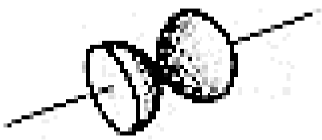
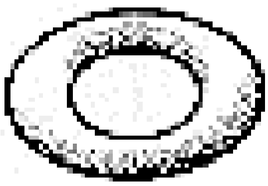
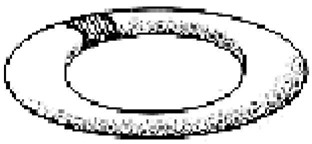
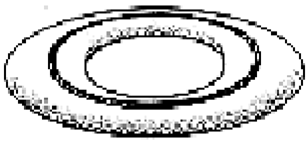
Definition	Sketch	Sources
Cylinder to any rectangle in plane perpendicular to cylinder axis.		Tripp, Hwang & Crank (1962) [48].
Finite area on exterior of cylinder to finite area on plane parallel to cylinder axis.		Stevenson & Grafton (1961) [47]. Report SID-61-91, North American Aviation (AFASD TR 61-119, pt. 1), 1961.
Cylinder to any rectangle in plane parallel to cylinder axis.		Tripp, Hwang & Crank (1962) [48]. Special Report 16, Kansas State University Bulletin, vol. 46, No. 4, 1962.

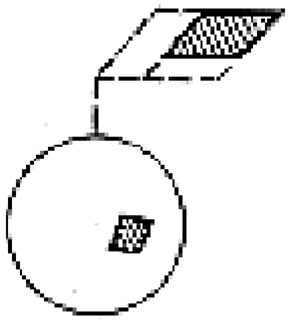
Definition	Sketch	Sources
Finite area on exterior of cylinder to finite area on skewed plane.		Stevenson & Grafton (1961) [47]. Report SID-61-91, North American Aviation (AFASD TR 61-119, pt. 1), 1961.
Outside surface of cylinder to perpendicular right triangle; triangle is in plane of cylinder base with one vertex of triangle at center of base.		Tripp, Hwang & Crank (1962) [48].
Cylinder and plane of equal length parallel to cylinder axis; plane inside cylinder; all factors between plane and inner surface of cylinder.		Leuenberger & Person (1956) [24].

Definition	Sketch	Sources
Finite areas on interior of right circular cylinder.		Stevenson & Grafton (1961) [47].
Finite area on exterior of cylinder to finite area on exterior of parallel cylinder.		Stevenson & Grafton (1961) [47]. Report SID-61-91, North American Aviation (AFASD TR 61-119, pt. 1), 1961.
Concentric cylinders of different radii, one atop other; factors between inside of upper cylinder and inside or base of lower cylinder.		Leuenberger & Person (1956) [24].

Definition	Sketch	Sources
Finite area on exterior of inner cylinder to finite area on interior of concentric outer cylinder.		Stevenson & Grafton (1961) [47].
Two tubes connected with fin to finite thickness; length can be finite or infinite; all factors between finite surfaces formulated in terms of integrations between differential strips.		Sotos & Stockman (1964) [39]. NASA TN D-2556, 1964.
Two tubes connected with tapered fins of finite thickness; tube length can be finite or infinite; all factors between finite surfaces formulated in terms of integrations between differential strips.		Sotos & Stockman (1964) [39].
Sandwich tube and fin structure of infinite or finite length; all factors between finite surfaces formulated in terms of integrations between differential strips.		Sotos & Stockman (1964) [39]. NASA TN D-2556, 1964.

Definition	Sketch	Sources
Concentric cylinders connected by fin of finite thickness; length finite or infinite; all factors between finite surfaces formulated in terms of integrations between differential strips.		Sotos & Stockman (1964) [39].
From Moebius strip to itself.		Stasenko (1967) [46]. Akad. Nauk SSSR, Izv. Energetika Transport, pp. 104-107, 1967.
Area on sphere to area on another sphere.		Grier (1969) [13]. NASA SP-3050, 1969 [28].
Band on one sphere to band on another sphere.		Grier (1969) [13].
Area on sphere to cap on another sphere.		Grier (1969) [13]. NASA SP-3050, 1969 [28].

Definition	Sketch	Sources
Cap on sphere to band on another sphere.		Grier (1969) [13].
Cap on sphere to cap on another sphere.		Grier (1969) [13].
Hemisphere to coaxial hemisphere in contact.		Wakao, Kato & Furuya (1969) [49]. Int. J. Heat Mass Transfer, vol. 12, No. 1, 1969, pp. 118-120.
Exterior of toroid to itself.		Sommers & Grier (1969) [38]. Journal of Heat Transfer, vol. 91, No. 3, 1969, pp. 459-461.
Segment of finite width on toroid to exterior of toroid.		Grier & Sommers (1969) [14]. NASA TN D-5006, 1969 [32].
Toroidal band of finite width to exterior of toroid.		Grier & Sommers (1969) [14].

Definition	Sketch	Sources
Area on surface of sphere to rectangle in plane perpendicular to axis of sphere.		Stevenson & Grafton (1961) [47]. Report SID-61-91, North American Aviation (AFASD TR 61-119, pt. 1), 1961.
Reference: Siegel & Howell (1972) [37].		

5 Specular surfaces

5.1 General

The specular view factor, F^s_{12}, between two specularly reflecting gray surfaces A_1 and A_2 is defined, (Perlmutter & Siegel (1963) [33]) as the fraction of the energy leaving diffusely the isothermal surface A_1 the impinges A_2, either directly or through any number of specular reflections from these or other gray surfaces of the whole system. Notice that a given amount of energy leaving A_1, if specularly reflected, may be counted several times on its arrival to A_2, so that F^s_{12} may be larger than one. On the other hand, the ratio of the energy absorbed by A_2 to that emitted by A_1 is smaller than one, depending its value, for a given geometrical system, on the specular reflectance of the surfaces forming the system.

The above definition of specular view factor shows some peculiarities that were not present in the definition of diffuse view factors. Such peculiarities are:

1. When calculating the radiant interchange between gray diffuse surfaces, the effect of the diffuse reflectance of the surfaces is not included in the view factor, since it is accounted for by means of the radiosity, B, which takes into account both the emitted and the diffusely reflected radiations.

   $B = \varepsilon\sigma T^4 + \rho^d H$,

   H represents the radiant flux incident on the emitting surfaces per unit time and unit area, and ρ^d is the diffuse reflectance. Thus $B_1A_1F_{12}$ is the heat arriving directly to A_2 from A_1 without being reflected in any surface after the last time it leaves A_1.

   On the other hand, when specular surfaces are involved, it is usual to leave unchanged the factor corresponding to diffuse radiosity, and to include specular reflections in the view factor F^s_{12}. In any case $B_1A_1F^s_{12}$ measures the heat arriving to A_2 from A_1 both directly and through all possible specular inter-reflections, but it should be emphasized that B_1 indicates the diffusely-tributed radiant flux leaving A_1 per unit time and unit area.

   A ray, that leaving A_1 reaches A_2 after a reflection from A_j, is accounted for as coming from A_j if this surface is diffuse reflecting and as coming from A_1 if A_j is specularly reflecting.

2. The concept of diffuse view factor involves only the emitting surface, A_1, and the receiving surface, A_2, while the parallel concept of specular view factor involves, in addition to A_1 and A_2, all partially or totally specular surfaces of the system where rays coming from A_1 can be reflected before reaching A_2.

3. It is obvious that, for a given geometrical system, the radiative transfer equation may be written in a unified fashion in terms of the temperature, optical characteristics of the surfaces, and view factors, no matter whether some of these surfaces are specular or not,

provided that the diffuse radiosity and the appropriate view factors are used for the computation.

For a system of N specular surfaces, having specular reflectances, ρ_j^s, the specular view factor, F^s_{12}, between two of them is:

$$F_{12}^{s} = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\cdots\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\left(\rho_1^s\right)^i\left(\rho_2^s\right)^j\left(\rho_3^s\right)^k\cdots\left(\rho_{N-1}^s\right)^p\left(\rho_N^s\right)^q K_{12}(i,j,k,\ldots,p,q) \quad [5\text{-}1]$$

with

$$\lim_{\rho_j^s \to 0}\left(\rho_j^s\right)^o = 1 \quad [5\text{-}2]$$

for every ρ_j^s.

The factor K_{12} may be interpreted as the fraction of the radiative energy leaving A_1 which reaches A_2 after i specular reflections from A_1, j from A_2, k from A_3,..., p from A_{N-1}, and q from A_N, under the assumption that there is no absorption, so that the mirrors do nothing except to change the direction of the impinging rays. Due to some geometrical constraints, several K factors may vanish.

A reasoning based on the fact that the reversal of the rays arriving to A_2 from A_1, after reflection from several surfaces of the system, is exactly equivalent to the system of the rays arriving to A_1 from A_2, after reflection from the same surfaces, indicates that, independently of what surfaces A_1 and A_2 are considered, their specular view factors satisfy the reciprocity relation:

$A_1 F^s_{12} = A_2 F^s_{21}$

When an enclosure of N surfaces, A_1, A_2, ..., A_N is considered, the specular view factors satisfy the relation:

$$\sum_{j=1}^{N}\left(1-\rho_j^s\right)F_{ij} = 1 \quad [5\text{-}3]$$

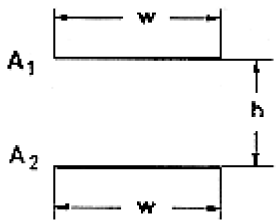
for any surface A_i. This relationship results from the fact that the overall heat transfer in the enclosure should be zero.

5.2 Two planar specular surfaces

5.2.1 Two-dimensional configurations

5.2.1.1 Parallel strips of equal width

Two infinitely long directly opposed parallel strips of same finite width.



$H = w/h$

$R = \rho_1^s \rho_2^s$

Formula:

$$F_{12}^s = F_{21}^s = K(1) + \sum_{n=1}^{\infty} R^n K(2n+1) \qquad [5\text{-}4]$$

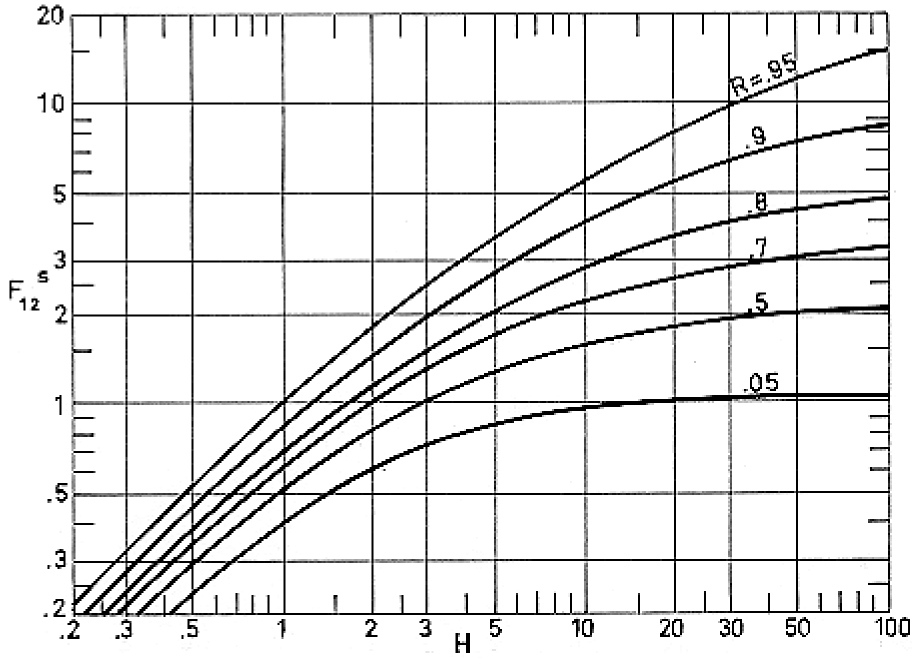
$$\frac{F_{11}^s}{\rho_2^s} = \frac{F_{22}^s}{\rho_1^s} = K(2) + \sum_{n=1}^{\infty} R^n K(2n+2) \qquad [5\text{-}5]$$

where:

$$K(m) = \frac{1}{H}\left(\sqrt{H^2 + m^2} - m\right) \qquad [5\text{-}6]$$

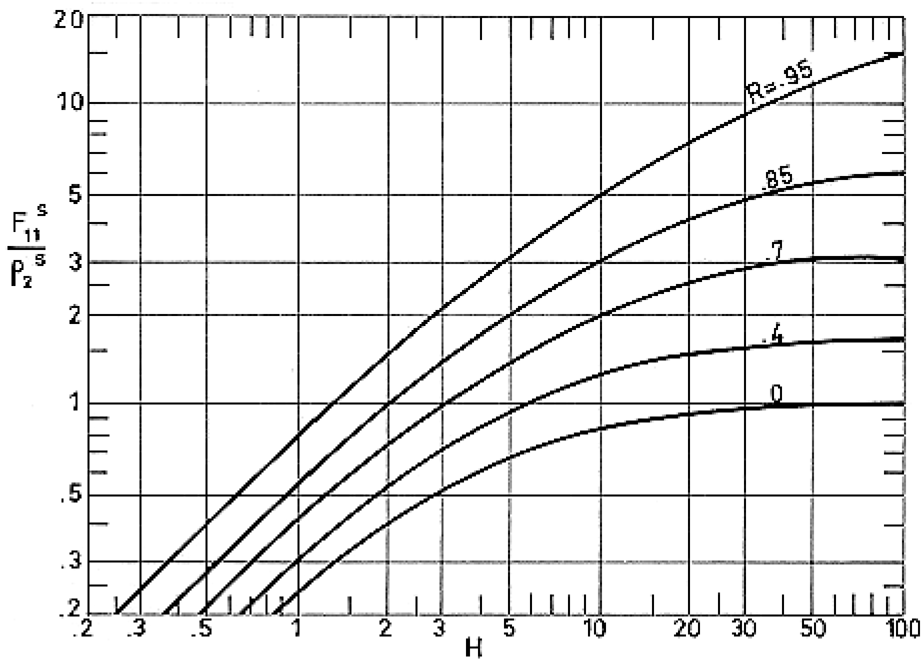
n is the number of specular reflections which a ray suffers before the receiving surface.

Reference: These expressions have been obtained by the compiler.



Note: non-si units are used in this figure

Figure 5-1: Values of F_{12} as a function of R and H. Calculated by the compiler.

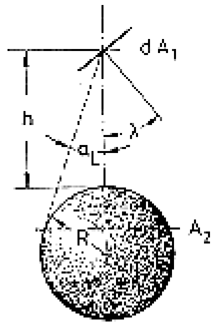


Note: non-si units are used in this figure

Figure 5-2: Values of F^{s}_{11}/ρ_2^{s} as a function of R and H. Calculated by the compiler.

5.2.1.2 Strips of equal width at any angle

Two infinitely long plates of equal finite width, w, having one common edge, and at an included angle ϕ to each other.



$R = \rho_1^s \rho_2^s$

Formula:

$$F_{12}^s = F_{21}^s = 1 - \sin\frac{\phi}{2} + \sum_{n=1}^{n<\frac{90}{\phi}-\frac{1}{2}} R^n \left[1 - \sin\left(n + \frac{1}{2}\right)\phi\right] \qquad [5\text{-}7]$$

$$\frac{F_{11}^s}{\rho_2^s} = \frac{F_{22}^s}{\rho_1^s} = 1 - \sin\phi + \sum_{n=1}^{n<\frac{90}{\phi}-1} R^n \left[1 - \sin(n+1)\phi\right] \qquad [5\text{-}8]$$

where n is the number of specular reflections which a ray suffers before reaching the receiving surface.

When $\phi \geq 60°$, the rays reach the second surface without suffering any specular reflection from the first. Thus

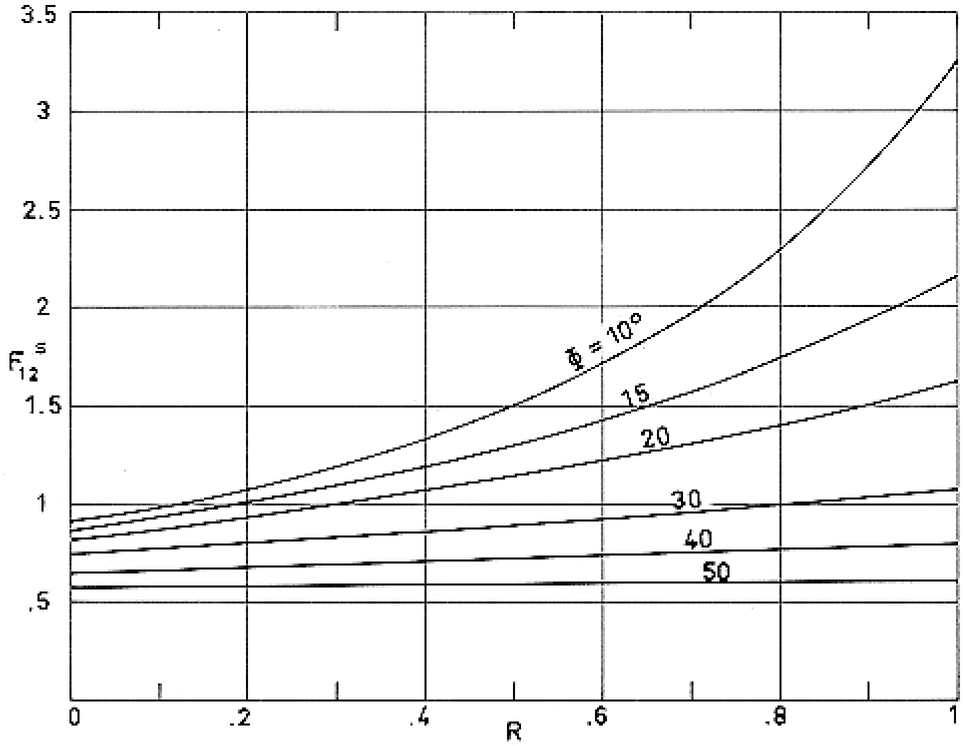
$$F_{12}^s = F_{21}^s = 1 - \sin\frac{\phi}{2} \qquad [5\text{-}9]$$

When $\phi \geq 45°$, the rays reach the first surface after suffering at most one specular reflection from the second. Thus

$$\frac{F_{11}^s}{\rho_2^s} = \frac{F_{22}^s}{\rho_1^s} = 1 - \sin\phi \qquad [5\text{-}10]$$

References: These expressions have been obtained by the compiler. The cases $\phi = 45°, 90°$ have been dealt with by Ecker & Sparrow (1961).

Notice that, when $\phi \geq 60°$, F_{12}^s is independent of R.



Note: non-si units are used in this figure

Figure 5-3: Values of F^s_{12} as a function of R for different values of ϕ. Calculated by the compiler.

Notice that when $\phi \geq 45°$, F^s_{11}/ρ_2^s is independent of R.

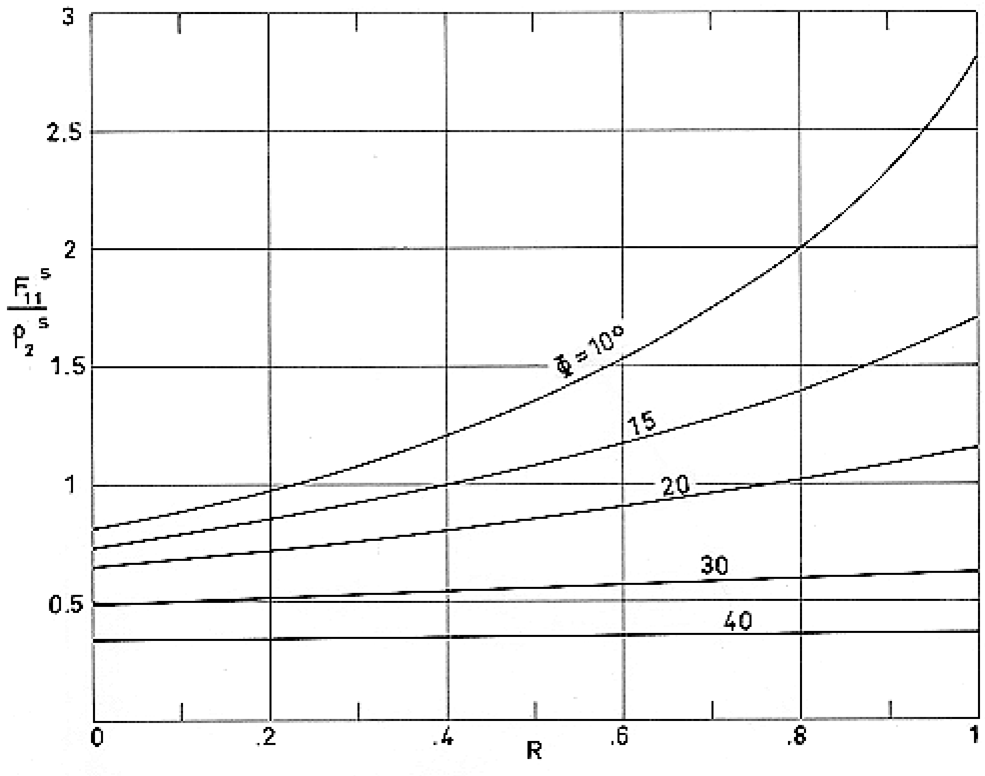
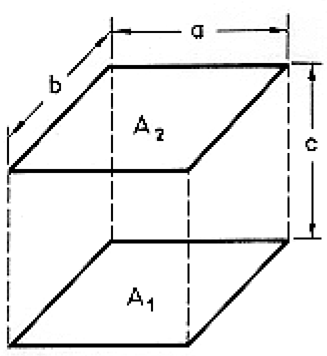


Figure 5-4: Values of F^s_{11}/ρ_2^s as a function of R for different values of ϕ. Calculated by the compiler.

5.2.2 Parallel, directly opposed rectangles of same width and length



$X = a/c$

$Z = b/a$

$R = \rho_1^s\rho_2^s$

Formula:

$$F_{12}^s = F_{21}^s = K(1) + \sum_{n=1}^{\infty} R^n K(2n+1) \quad [5\text{-}11]$$

$$\frac{F_{11}^{s}}{\rho_2^{s}} = \frac{F_{22}^{s}}{\rho_1^{s}} = K(2) + \sum_{n=1}^{\infty} R^{n} K(2n+2) \qquad [5\text{-}12]$$

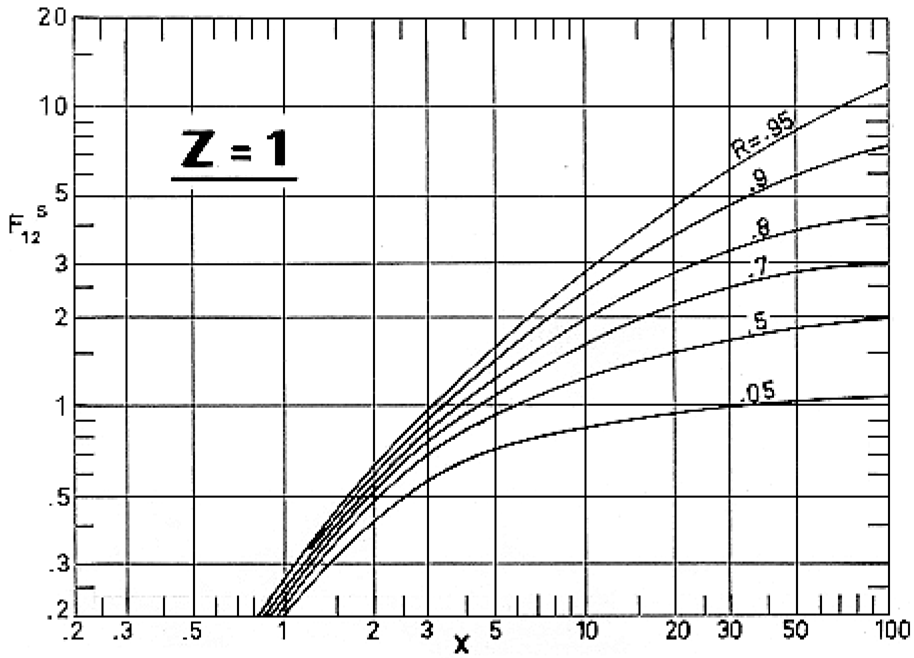
where

$$K(m) = \frac{2m^2}{\pi X^2 Z}\left\{ \begin{array}{l} \ln\left[\dfrac{\left[1+(X/m)^2\right]\left[1+(XZ/m)^2\right]}{1+(X/m)^2+(XZ/m)^2}\right]^{1/2} + \\ + \dfrac{XZ}{m}\left[\sqrt{1+(X/m)^2}\,\tan^{-1}\dfrac{XZ/m}{\sqrt{1+(X/m)^2}} - \tan^{-1}\dfrac{XZ}{m}\right] + \\ + \dfrac{X}{m}\left[\sqrt{1+(XZ/m)^2}\,\tan^{-1}\dfrac{X/m}{\sqrt{1+(XZ/m)^2}} - \tan^{-1}\dfrac{X}{m}\right] \end{array} \right\} \qquad [5\text{-}13]$$

Note: non-si units are used in this figure

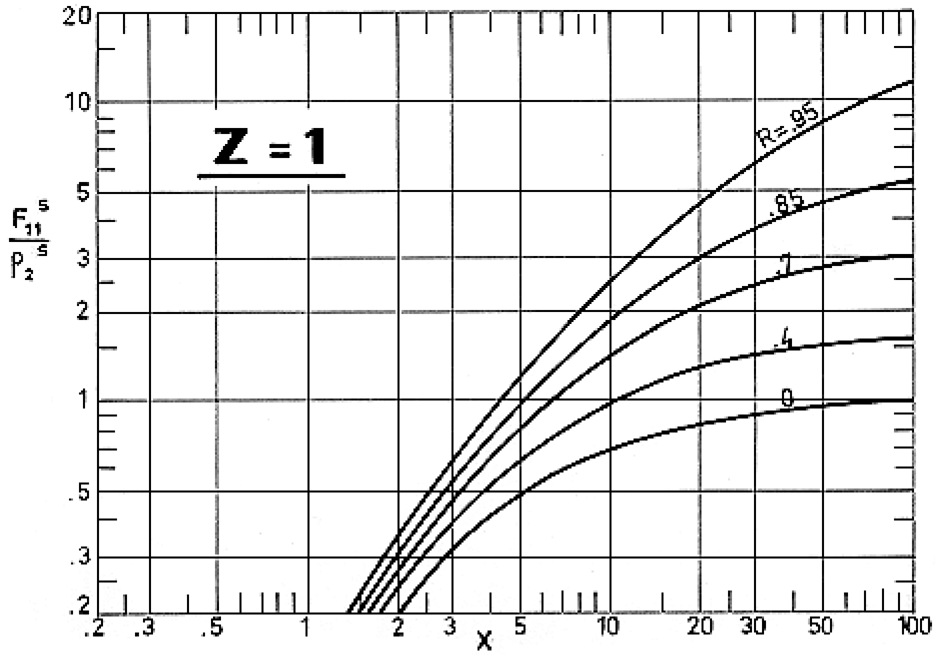
n is the number of specular reflections which a ray suffers before reaching the receiving surface.

References: These expressions have been obtained by the compiler.



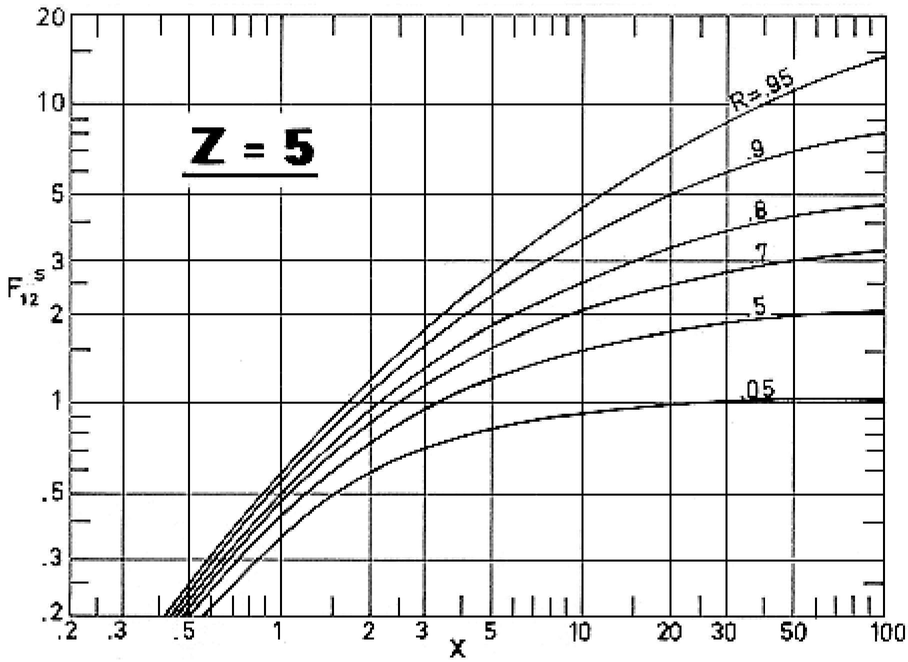
Note: non-si units are used in this figure

Figure 5-5: Values of F^{s}_{12} as a function of *R* and *X* for *Z* = 1. Calculated by the compiler.



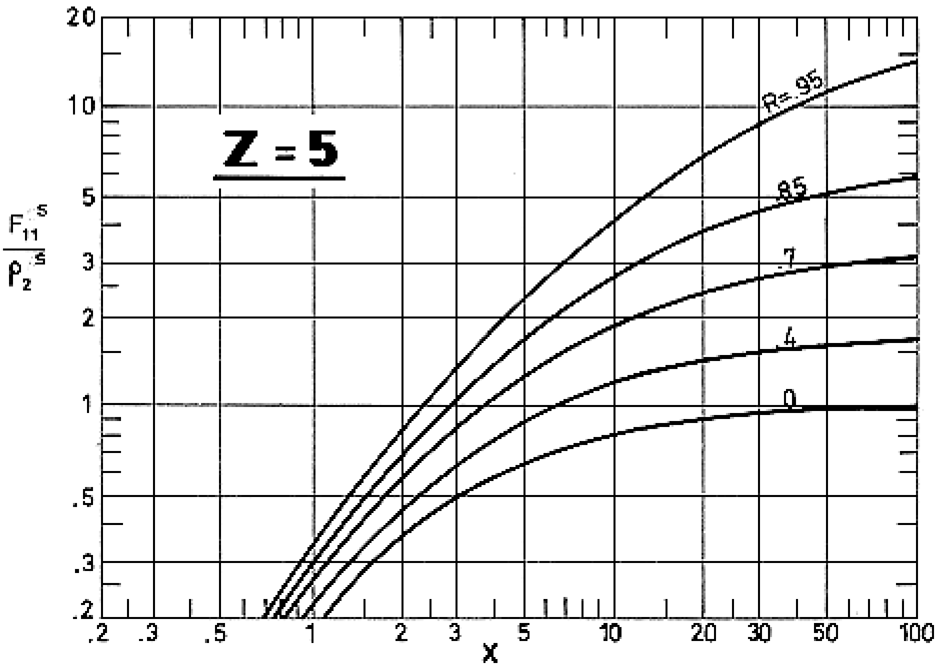
Note: non-si units are used in this figure

Figure 5-6: Values of F^s_{11}/ρ_2^s as a function of R and X for Z = 1. Calculated by the compiler.



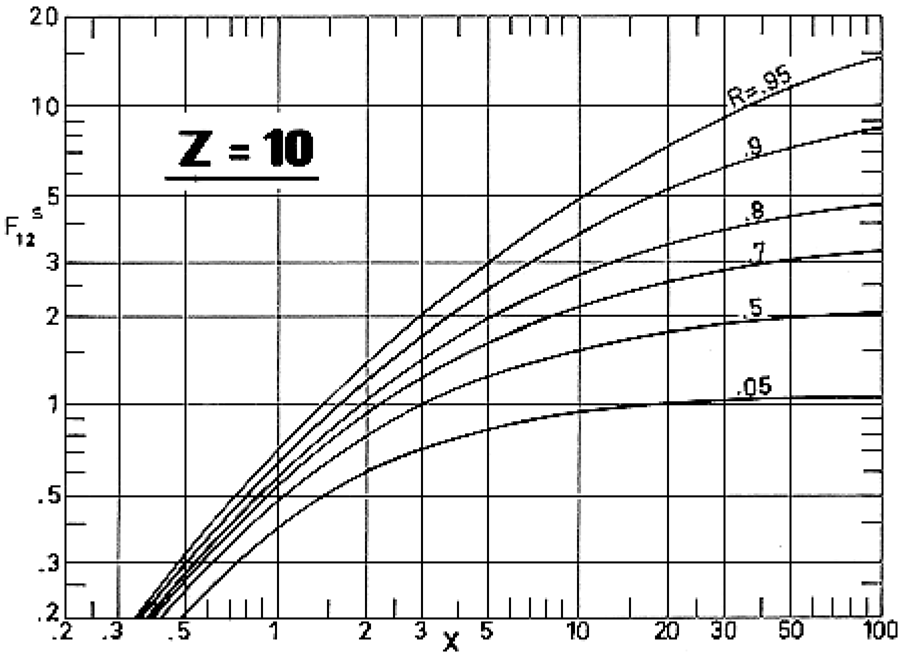
Note: non-si units are used in this figure

Figure 5-7: Values of F^s_{12} as a function of R and X for Z = 5. Calculated by the compiler.



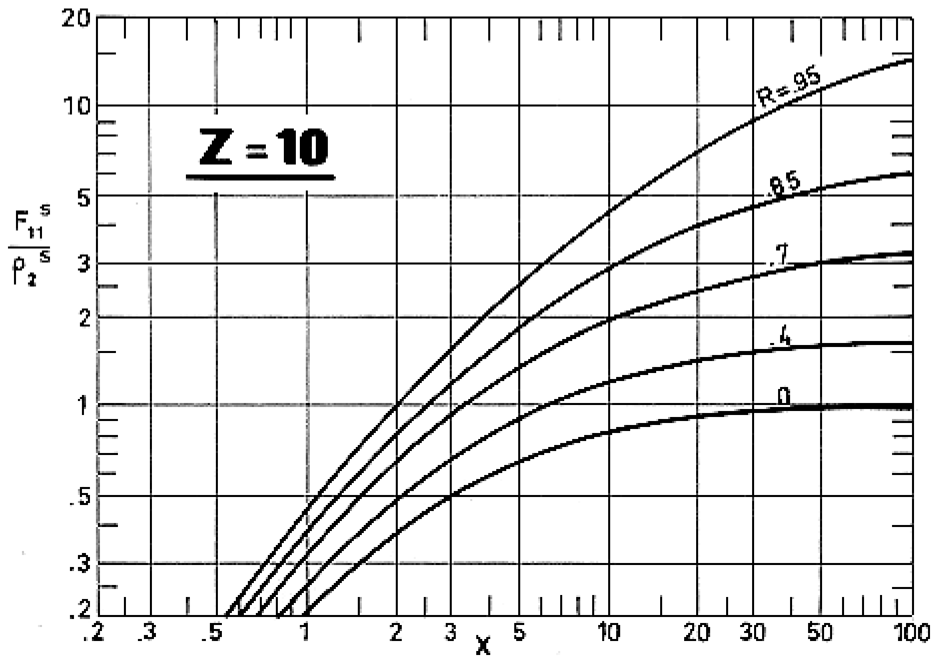
Note: non-si units are used in this figure

Figure 5-8: Values of F^s_{11}/ρ_2^s as a function of R and X for Z = 5. Calculated by the compiler.



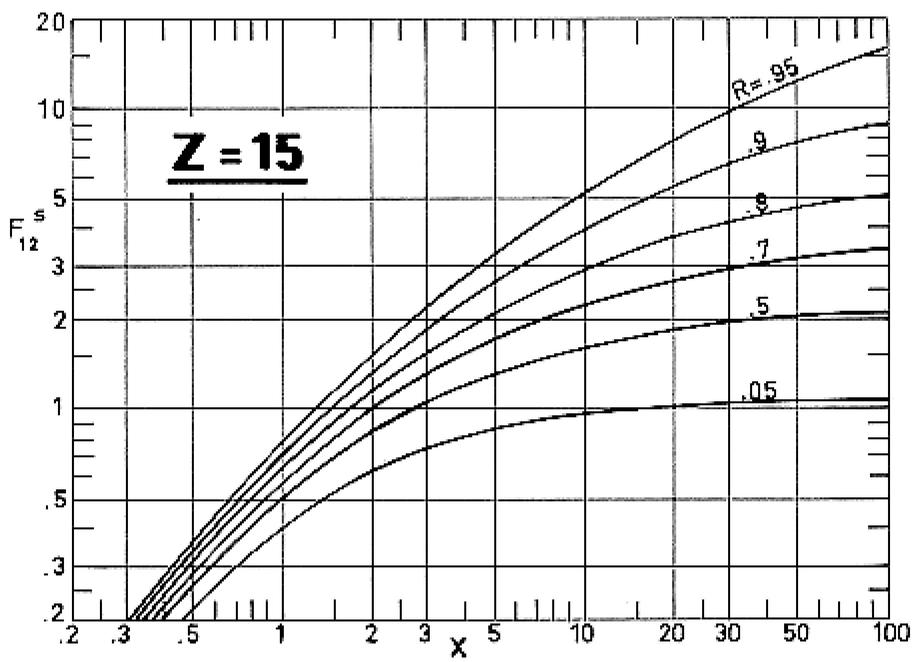
Note: non-si units are used in this figure

Figure 5-9: Values of F^s_{12} as a function of R and X for Z = 10. Calculated by the compiler.



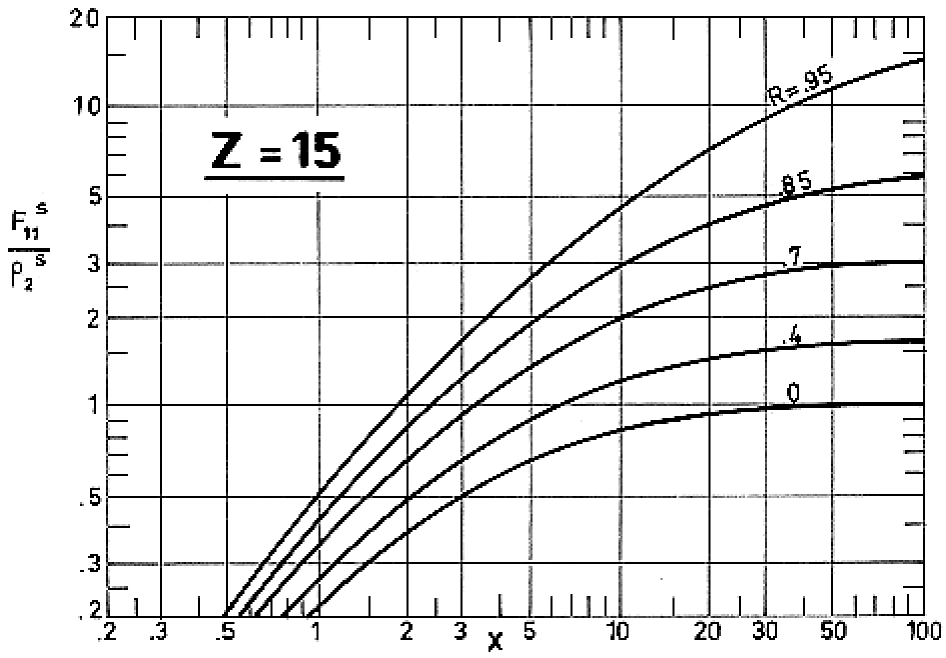
Note: non-si units are used in this figure

Figure 5-10: Values of F^s_{11}/ρ_2^s as a function of R and X for $Z = 10$. Calculated by the compiler.



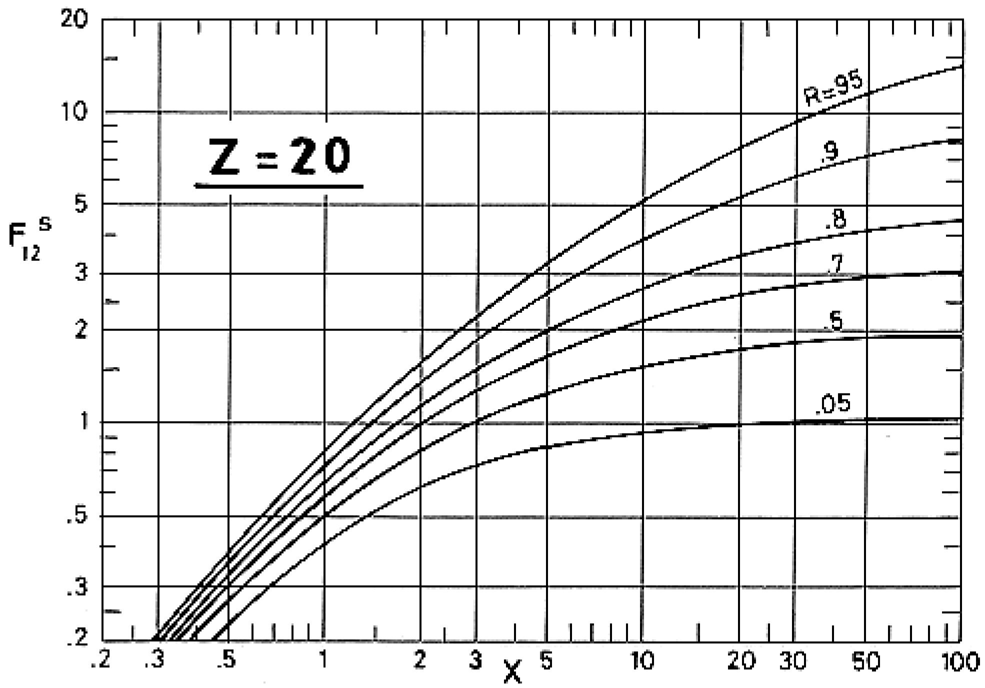
Note: non-si units are used in this figure

Figure 5-11: Values of F^s_{12} as a function of R and X for $Z = 15$. Calculated by the compiler.



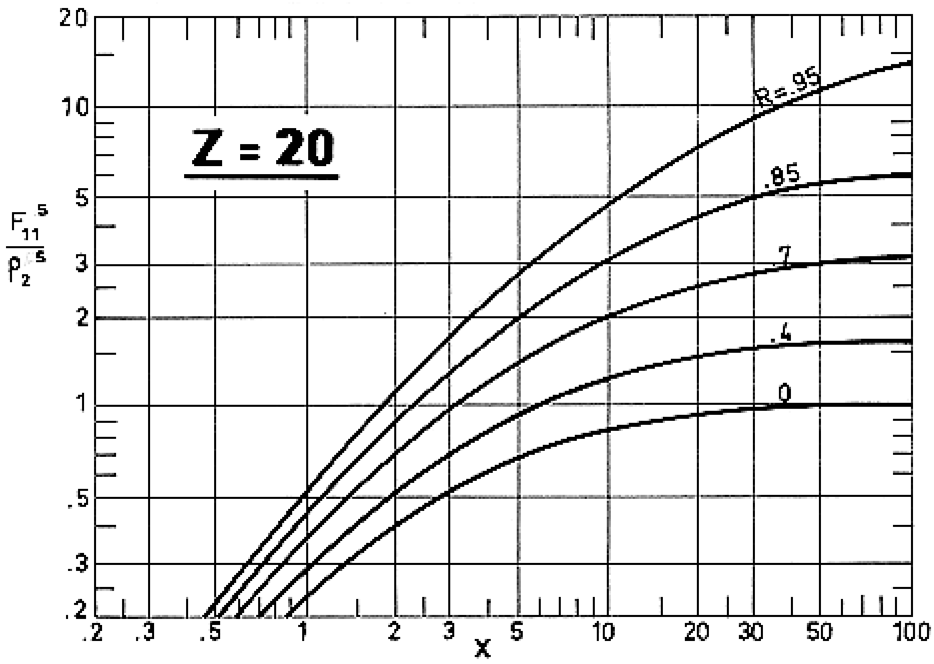
Note: non-si units are used in this figure

Figure 5-12: Values of F^s_{11}/ρ_2^s as a function of R and X for $Z = 15$. Calculated by the compiler.



Note: non-si units are used in this figure

Figure 5-13: Values of F^s_{12} as a function of R and X for $Z = 20$. Calculated by the compiler.

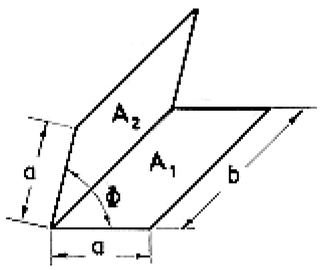


Note: non-si units are used in this figure

Figure 5-14: Values of F^s_{11}/ρ_2^s as a function of R and X for Z = 20. Calculated by the compiler.

5.2.3 Rectangles of same width and length with one common edge

Two specular finite rectangles of the same dimensions having a common edge and at an included angle ϕ.



$L = a/b$

$R = \rho_1^s \rho_2^s$

Formula:

$$F_{12}^s = F_{21}^s = K(1) + \sum_{n=1}^{n<\frac{1}{2}\left(\frac{180}{\phi}-1\right)} R^n K(2n+1) \qquad [5\text{-}14]$$

Note: non-si units are used in this figure

$$\frac{F_{11}^{s}}{\rho_2^{s}} = \frac{F_{22}^{s}}{\rho_1^{s}} = K(2) + \sum_{n=1}^{n<\frac{1}{2}\left(\frac{180}{\phi}-1\right)} R^{n} K(2n+2) \qquad [5\text{-}15]$$

Note: non-si units are used in this figure

In these expressions *K(m)* is the diffuse view factor between two rectangles having the same width and length of that considered and at an included angle $m\phi$. The value of this diffuse view factor has been obtained by using the results by Feingold (1966) [11].

n is the number of specular reflections which a ray suffers before reaching the receiving surface.

Comments:

When $\phi \geq 60°$, the rays reaching the second surface did not suffer any specular reflection by the first.

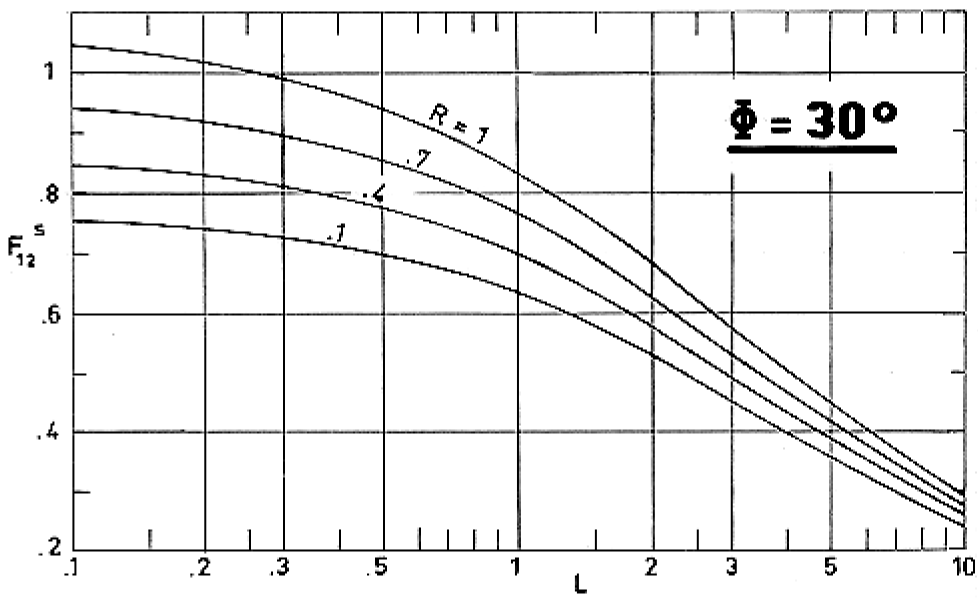
$F^{s}_{12} = F^{s}_{21} = K(1)$.

When $\phi \geq 45°$, the rays reaching the first surface did not suffer more than one specular reflection by the second.

$$\frac{F_{11}^{s}}{\rho_2^{s}} = \frac{F_{22}^{s}}{\rho_1^{s}} = K(2) \qquad [5\text{-}16]$$

Note: non-si units are used in this figure

Reference: These expressions have been obtained by the compiler.



Note: non-si units are used in this figure

Figure 5-15: Values of F^{s}_{12} vs. aspect ratio, *L*, for different values of *R*. ϕ = 30°. Calculated by the compiler.

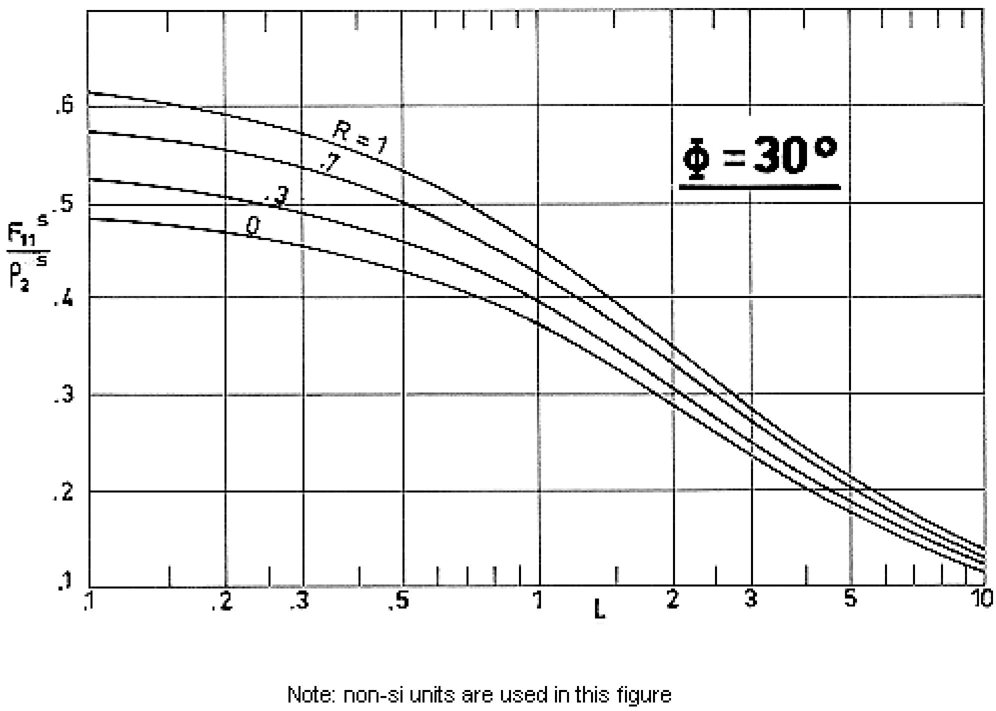


Figure 5-16: Values of F^s_{11}/ρ_2^s vs. aspect ratio, L, for different values of R . $\phi = 30°$. Calculated by the compiler.

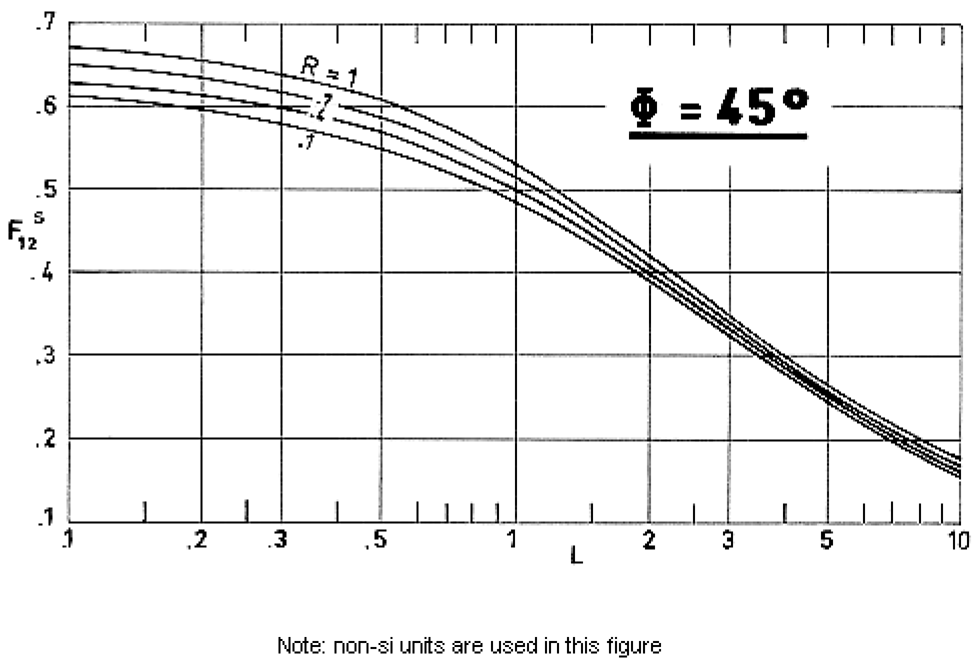
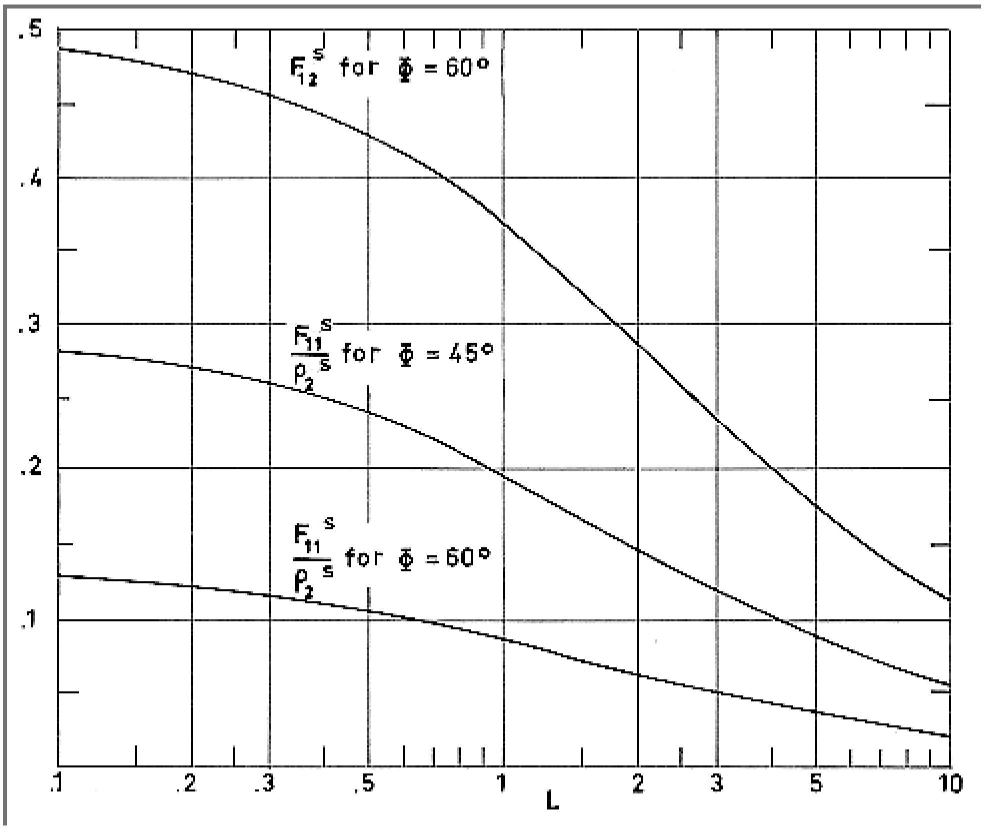


Figure 5-17: Values of F^s_{12} vs. aspect ratio, L, for different values of R. $\phi = 45º$. Calculated by the compiler.

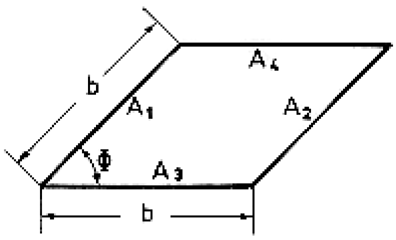


Note: non-si units are used in this figure

Figure 5-18: Values of F^s_{12} and F^s_{11}/ρ_2^s vs. aspect ratio, L, for the limiting values of ϕ. Calculated by the compiler.

5.3 Planar specular and planar diffuse surface

5.3.1 Two dimensional cavities. Cylinders of quadrangular cross section



Specular view factors between the different inner surfaces of a cylinder of quadrangular cross section formed by two specular and two diffuse parallel strips.

The surfaces A_1 and A_2 are specular-reflecting while A_3 and A_4 are diffuse reflecting.

$\rho^s = \rho_1^s \rho_2^s$

Formula:

$$\begin{aligned} F_{11}^{s} = F_{22}^{s} = & \sum_{n=0}^{n=K-1} \left(\rho^{s}\right)^{2n+1} \left[\sqrt{(2n+2)^{2} \sin^{2}\Phi + 1} - (2n+1)\sin\Phi - 1 \right] + \\ & + \sum_{n=K}^{\infty} \left(\rho^{s}\right)^{2n+1} \left[\sqrt{(1-\cos\Phi)^{2} + (2n+1)^{2}\sin^{2}\Phi} - (2n+1)\sin\Phi \right] \end{aligned} \qquad [5\text{-}17]$$

Note: non-si units are used in this figure

$$\begin{aligned} F_{12}^{s} = F_{21}^{s} = & \sum_{n=0}^{n=K} \left(\rho^{s}\right)^{2n} \left[\frac{\sqrt{(1+\cos\Phi)^{2} + (2n+1)^{2}\sin^{2}\Phi} + \sqrt{(1-\cos\Phi)^{2} + (2n+1)^{2}\sin^{2}\Phi}}{2} - 1 - 2n\sin\Phi \right] + \\ & + \sum_{n=K+1}^{\infty} \left(\rho^{s}\right)^{2n} \left[\frac{\sqrt{(1-\cos\Phi)^{2} + (2n+1)^{2}\sin^{2}\Phi} + \sqrt{(1-\cos\Phi)^{2} + (2n-1)^{2}\sin^{2}\Phi}}{2} \right] \end{aligned} \qquad [5\text{-}18]$$

Note: non-si units are used in this figure

$$\begin{aligned} F_{31}^{s} = F_{13}^{s} = F_{42}^{s} = F_{24}^{s} = & \left[1 - \frac{\sqrt{(1-\cos\Phi)^{2} + \sin^{2}\Phi}}{2} \right] + \\ & + \sum_{n=1}^{n=K} \left\{ \begin{aligned} & \left(\rho^{s}\right)^{2n-1} \left[\frac{\sqrt{(1-\cos\Phi)^{2} + (2n-1)^{2}\sin^{2}\Phi} + 1 - \sqrt{(2n)^{2}\sin^{2}\Phi + 1}}{2} \right] + \\ & + \left(\rho^{s}\right)^{2n} \left[\frac{\sqrt{(2n)^{2}\sin^{2}\Phi + 1} - 1 - \sqrt{(1-\cos\Phi)^{2} + (2n+1)^{2}\sin^{2}\Phi}}{2} \right] \end{aligned} \right\} + \\ & + \sum_{n=K+1}^{\infty} \left(\rho^{s}\right)^{2n} \left[\frac{\sqrt{(1-\cos\Phi)^{2} + (2n-1)^{2}\sin^{2}\Phi} - \sqrt{(1-\cos\Phi)^{2} + (2n+1)^{2}\sin^{2}\Phi}}{2} + \sin\Phi \right] \end{aligned} \qquad [5\text{-}19]$$

Note: non-si units are used in this figure

$$\begin{aligned} F_{32}^{s} = F_{23}^{s} = F_{41}^{s} = F_{14}^{s} = & \sum_{n=0}^{n=K} \left(\rho^{s}\right)^{2n} \left[\frac{\sqrt{1 + (2n)^{2}\sin^{2}\Phi} + 1 - \sqrt{(1-\cos\Phi)^{2} + (2n+1)^{2}\sin^{2}\Phi}}{2} \right] + \\ & + \sum_{n=0}^{K-1} \left(\rho^{s}\right)^{2n+1} \left[\frac{\sqrt{(1+\cos\Phi)^{2} + (2n+1)^{2}\sin^{2}\Phi} - 1 - \sqrt{(2n+2)^{2}\sin^{2}\Phi + 1}}{2} + \sin\Phi \right] + \\ & + \sum_{n=K+1}^{\infty} \left\{ \begin{aligned} & \left(\rho^{s}\right)^{2n+1} \left[\frac{\sqrt{(2n-1)^{2}\sin^{2}\Phi + (1-\cos\Phi)^{2}} - \sqrt{(1-\cos\Phi)^{2} + (2n+1)^{2}\sin^{2}\Phi}}{2} + \sin\Phi \right] + \\ & + \left(\rho^{s}\right)^{2n+1} \left[\frac{\sqrt{(1+\cos\Phi)^{2}(2n+1)^{2}\sin^{2}\Phi} - \sqrt{(1-\cos\Phi)^{2} + (2n+1)^{2}\sin^{2}\Phi}}{2} + \sin\Phi - 1 \right] \end{aligned} \right\} \end{aligned} \qquad [5\text{-}20]$$

Note: non-si units are used in this figure

In all these expressions K is the largest integer which is smaller than $1/(2\cos\Phi)$.

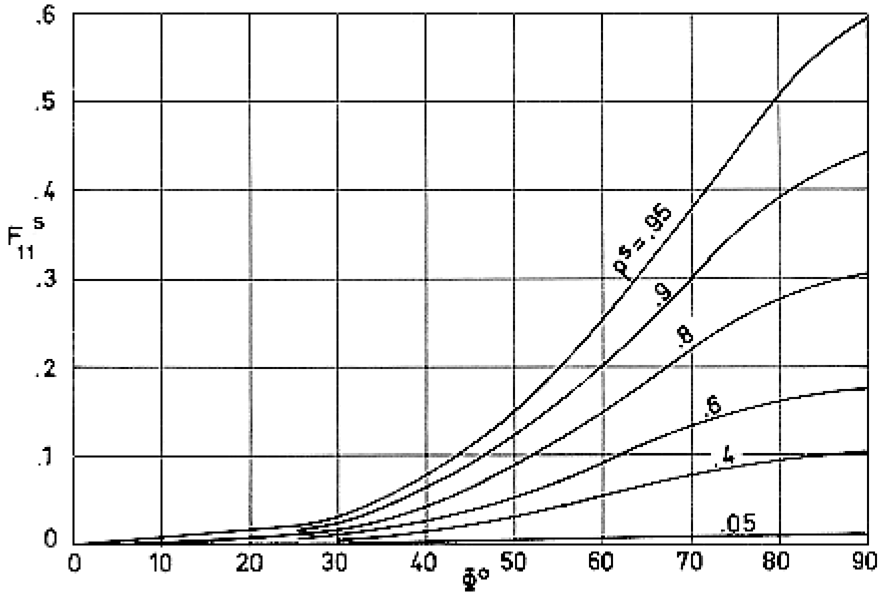
$$F_{33}^{s} = F_{44}^{s} = \rho^{s}\left(1 - \sin\Phi\right) \quad [5\text{-}21]$$

Note: non-si units are used in this figure

$$F_{34}^{s} = F_{43}^{s} = 1 - \left(1 - \rho^{s}\right)\left(F_{31}^{s} + F_{32}^{s}\right) - F_{33}^{s} \quad [5\text{-}22]$$

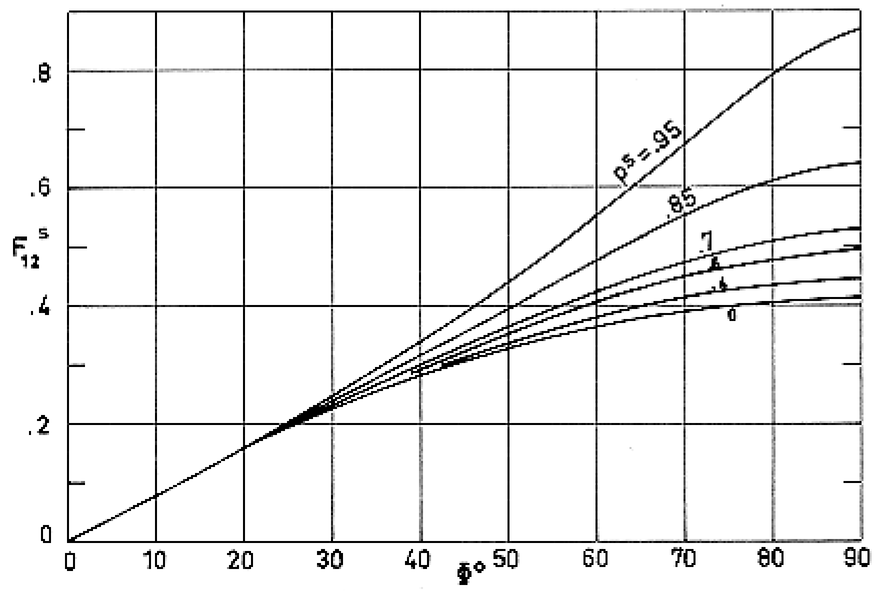
Note: non-si units are used in this figure

Reference: These expressions have been obtained by the compiler.



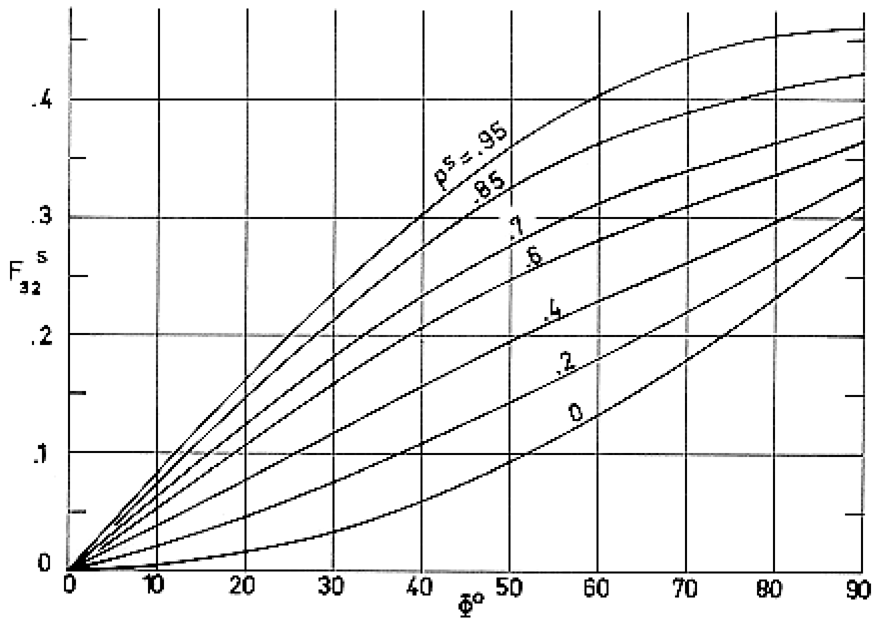
Note: non-si units are used in this figure

Figure 5-19: Values of F^{s}_{11} vs. ϕ for different values of the specular reflectance, ρ^{s}. Calculated by the compiler.



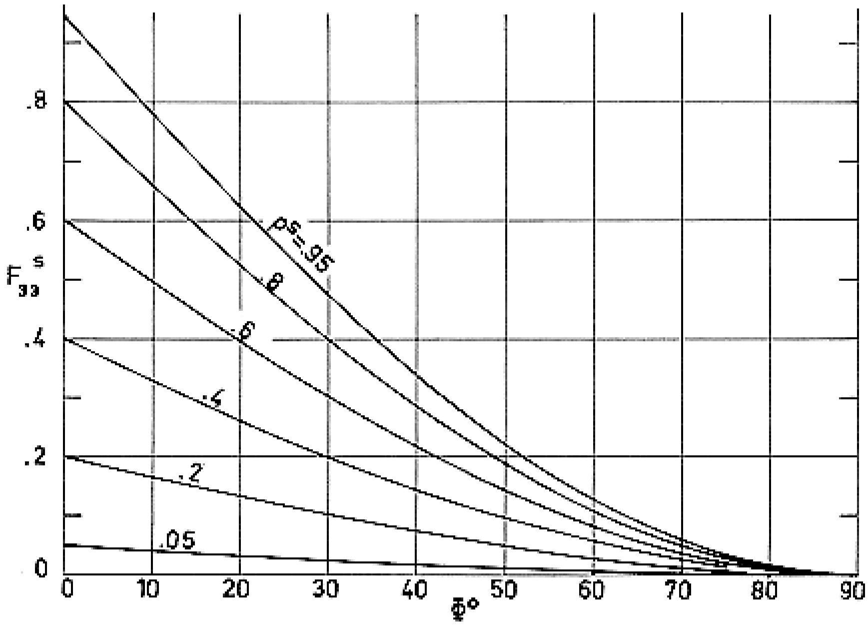
Note: non-si units are used in this figure

Figure 5-20: Values of F^{s}_{12} vs. ϕ for different values of the specular reflectance, ρ^{s}. Calculated by the compiler.



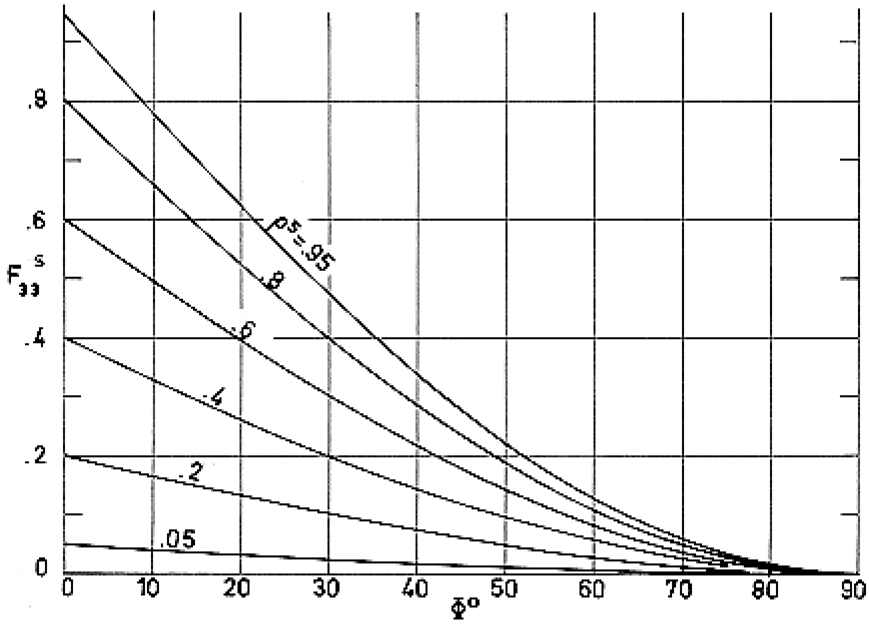
Note: non-si units are used in this figure

Figure 5-21: Values of F^{s}_{31} vs. ϕ for different values of the specular reflectance, ρ^{s}. Calculated by the compiler.



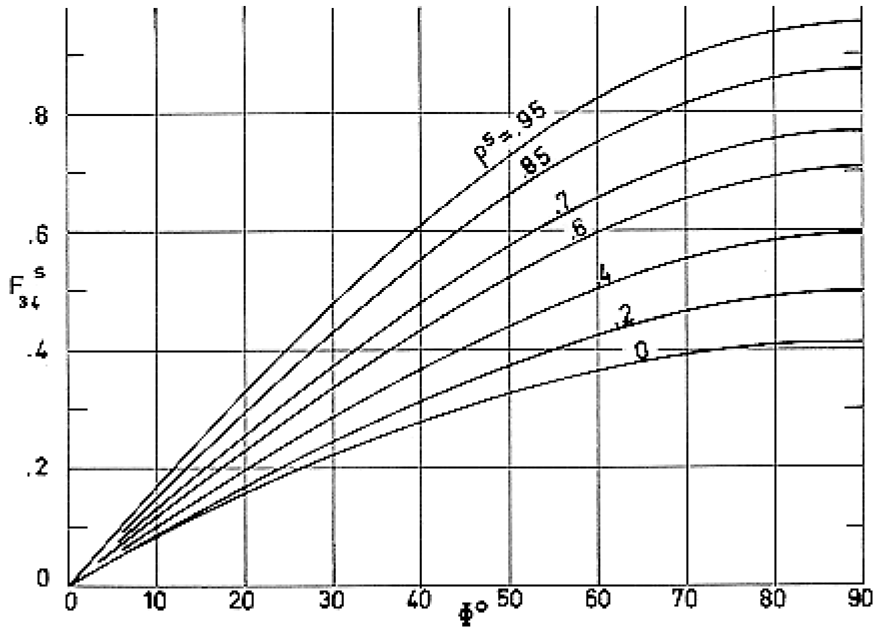
Note: non-si units are used in this figure

Figure 5-22: Values of $F_{32}{}^{s}$ vs. ϕ for different values of the specular reflectance, ρ^s. Calculated by the compiler.



Note: non-si units are used in this figure

Figure 5-23: Values of $F^s{}_{33}$ vs. ϕ for different values of the specular reflectance, ρ^s. Calculated by the compiler.



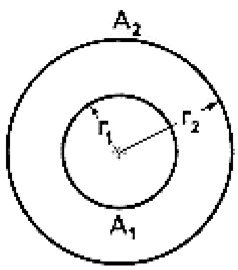
Note: non-si units are used in this figure

Figure 5-24: Values of F^{s}_{34} vs. ϕ for different values of the specular reflectance, ρ^{s}. Calculated by the compiler.

5.4 Non-planar specular surfaces

5.4.1 Concentric cylinder or concentric spheres

Concentric cylinders of infinite length, or concentric spheres.



Formula:

$$F_{11}^{s} = \frac{\rho_2^{s}}{1-\rho_1^{s}\rho_2^{s}} \quad [5\text{-}23]$$

$$F_{12}^{s} = \frac{1}{1-\rho_1^{s}\rho_2^{s}} \quad [5\text{-}24]$$

Note: non-si units are used in this figure

$$F_{21}^{s} = \frac{A_1 / A_2}{1 - \rho_1^s \rho_2^s} \qquad [5\text{-}25]$$

Note: non-si units are used in this figure

$$F_{22}^{s} = \left(1 - \frac{A_1}{A_2}\right)\frac{1}{1 - \rho_2^s} + \rho_1^s F_{21}^{s} \qquad [5\text{-}26]$$

where $A_1/A_2 = r_1/r_2$ for concentric cylinders, and $A_1/A_2 = (r_1/r_2)^2$ for concentric spheres.

Reference: These expressions have been obtained by the compiler.

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