Standardization training program
E-60 discipline: Control

Control Performance Standard
E-ST-60-10C
& ESA Pointing Error Engineering Handbook
ESSB-HB-E-003 Issue 1

& ESA Pointing Error Engineering Tool (PEET)

European Space Agency

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Standardization training program
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Control Performance Standard

E-ST-60-10C

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The ECSS E60 branch

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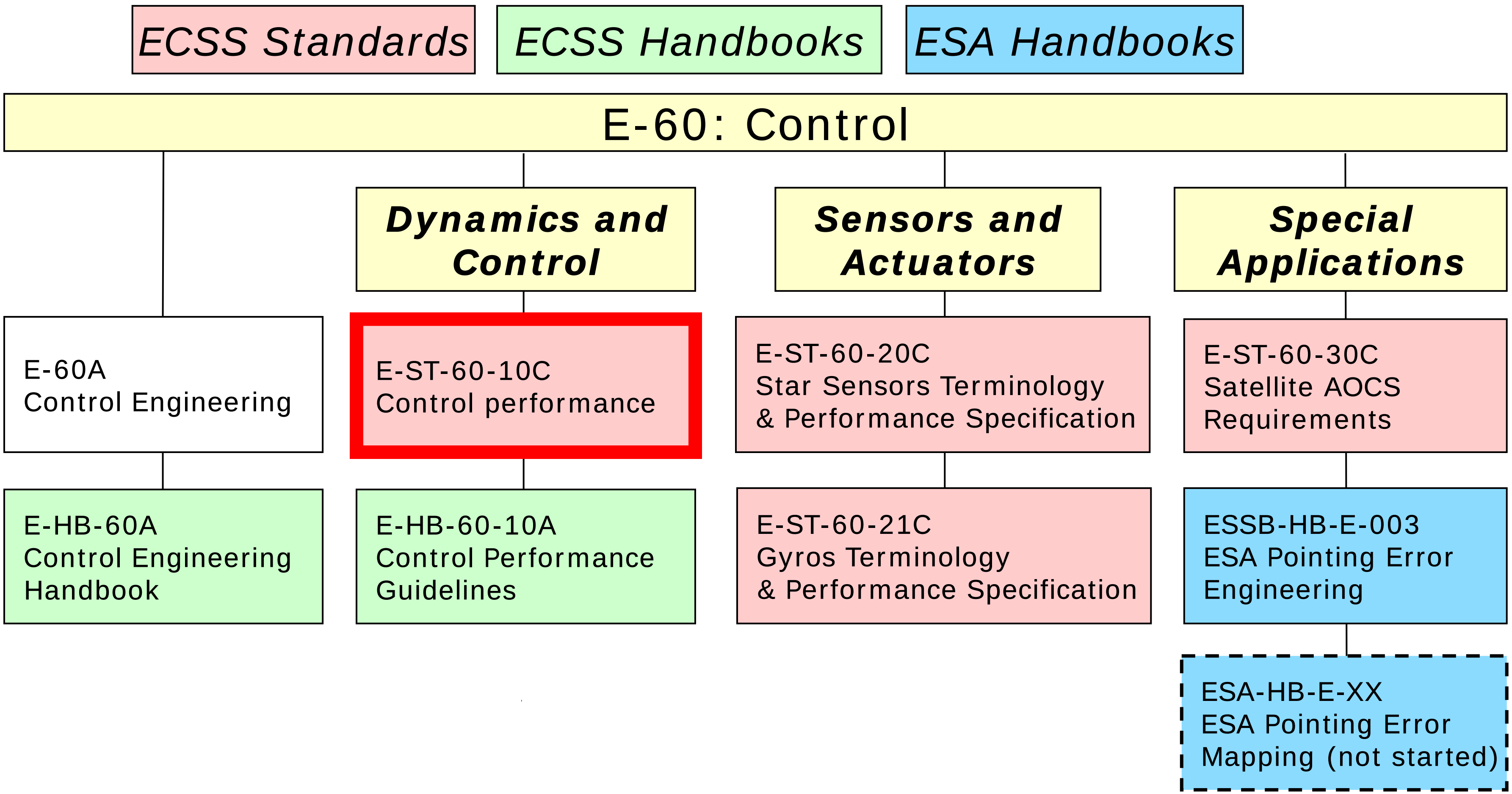

Introduction

Standardization training program
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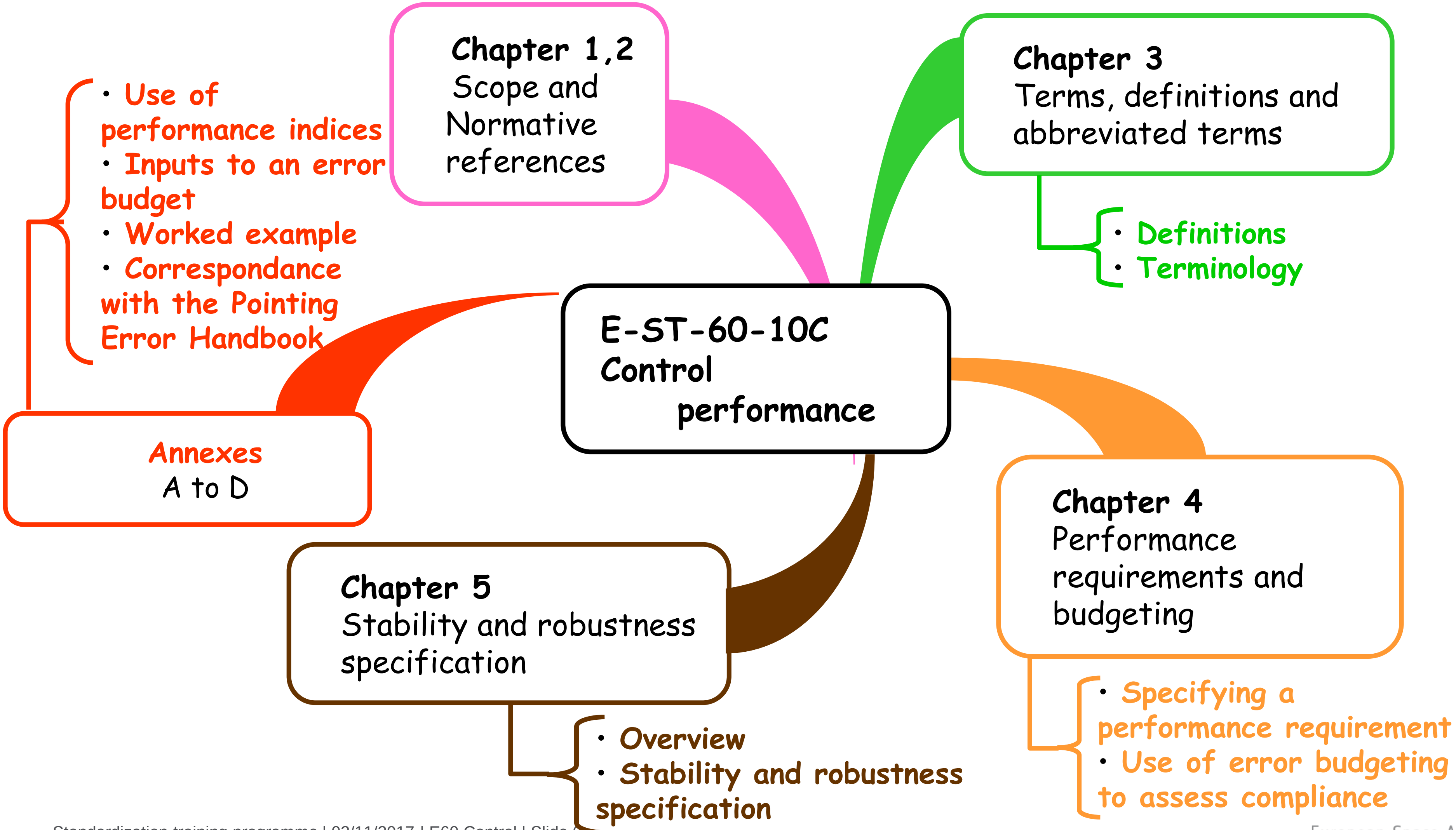
- The standard addresses the issue of control performance, in terms of definition, specification, verification and validation methods and processes.
  - It defines a general framework for handling **performance indicators**, which applies to all disciplines involving control engineering, at different levels ranging from equipment to system level.
  - It also focuses on the specific performance indicators applicable to the case of closed-loop control systems – mainly **stability and robustness**.
- Rules are provided for combining different error sources to build a **performance error budget** and use it to assess the **compliance** with a requirement.
  - Complementary material and guidelines for a step-by-step procedure for pointing error budgeting can be found in ECSS-E-HB-60-10A (Control Performance Guidelines) and in ESSB-E-HB-003 (Pointing Error Engineering Handbook)
- *The pointing accuracy shall be 0.1°* is **NOT** a good requirement. (worse yet: *The accuracy shall be as good as possible*)

Table of contents

Standardization training program E60 discipline: Control



Quick insight

Standardization training program E60 discipline: Control

- Chapter 3 gives the definition of control performance-related terms
  - Absolute knowledge error (AKE), absolute performance error (APE), error, error index, individual error source, knowledge error, ...
- Chapter 4 treats performance specifications and rules for error budgeting
  - The main features of performance requirements specification are defined together with the fundamental rules for identification, characterisation, and combination of contributors of an error budget.
- Chapter 5 treats stability and robustness specification and verification
  - Stability and robustness concepts for linear systems are introduced and requirements for their specification and verification are defined
- Annex A provides guidelines on using performance error indices
  - Statistical elements are provided that are useful in formulating error requirements and using and building an error budget
- Annex B discusses common types of errors and rules to budget for them
  - Formulas (some exact, some approximate) are provided to show how typical errors are budgeted for depending on the index and statistical interpretation

Ch. 4 - Stability and robustness specification

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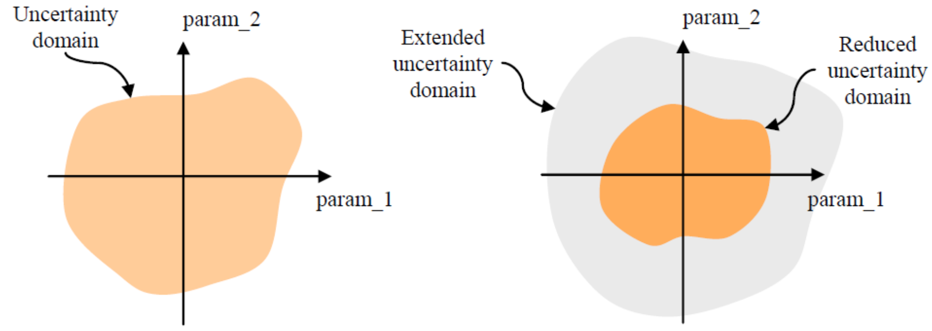
E60 discipline: Control

- Terminology (section 3)
  - Stability: ability of a system submitted to bounded external disturbances to remain indefinitely in a bounded domain around an equilibrium position or around an equilibrium trajectory
  - Performance: the system output meets the accuracy requirements
  - Robustness: ability of a controlled system to maintain some performance or stability characteristics in the presence of plant, sensors, actuators and/or environmental uncertainties
  - Stability margin: maximum excursion of some parameters describing a given control system for which the system remains stable
- The state-of-the-art for stability specification is not fully satisfactory.
  - A traditional rule exists, going back to the times of analogue controllers, asking for a gain margin > 6 dB, and a phase margin > 30°.
- A more consistent method is proposed for specifying stability and robustness (section 5.2)
  - For SISO loops **the gain margin, the phase margin and the modulus margin** shall be used as default indicators.
  - For MIMO loops use **sensitivity (disturb.) and complementary sensitivity (noise) functions**
  - If other indicators are selected by the supplier, this deviation shall be justified and the relationship with the default ones be established.

Ch. 4 - Stability and robustness verification

- In order to handle uncertainties (see the previous definition of robustness), **uncertainty domains** must be specified that consist of:
  - A list of uncertain parameters
  - For each of these parameters, an interval of uncertainty (or a dispersion) around the nominal value (in percentage or based on a statistical distribution)
  - When relevant, the root cause for the uncertainty (normally lack of characterisation, errors in parameters measurements, changes over life)
- A single uncertainty domain or reduced/extended domains may be defined
  - Nominal stability margins shall be demonstrated over the entire uncertainty domain (if a single domain is defined – section 5.2.6) or over the reduced domain (if reduced and extended domains are defined - section 5.2.7).
  - Degraded stability margins shall be demonstrated over the extended domain (section 5.2.7).



Ch. 5 - Terminology – Pointing Error Indices

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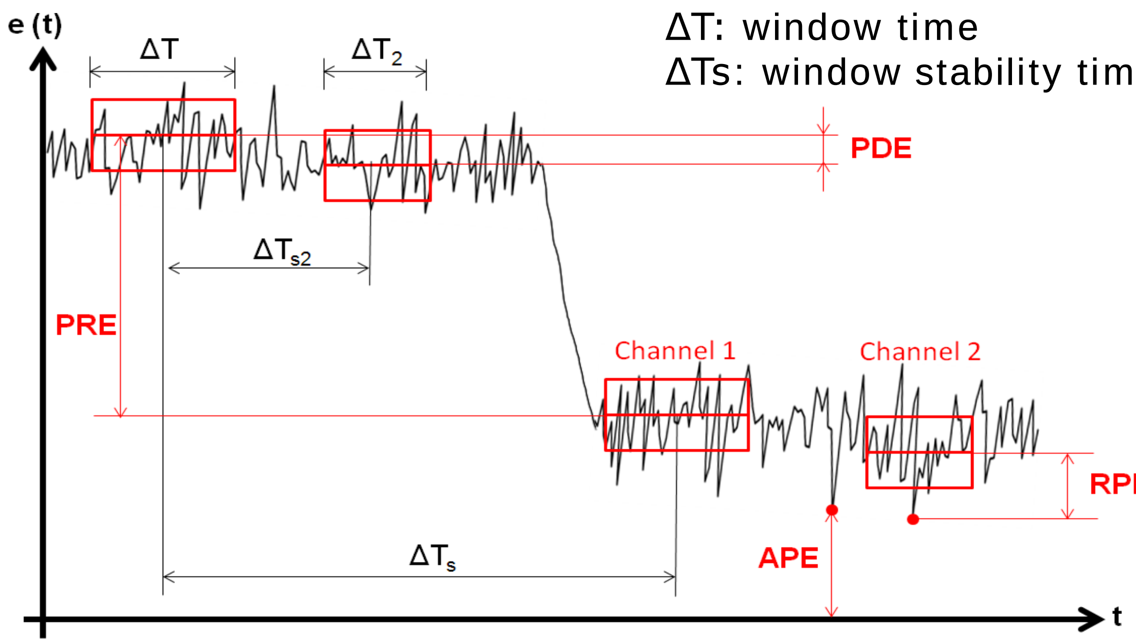
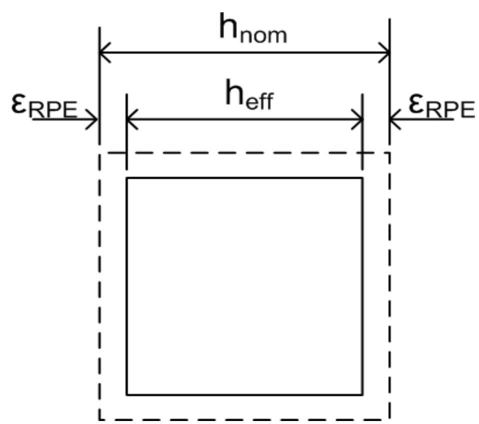
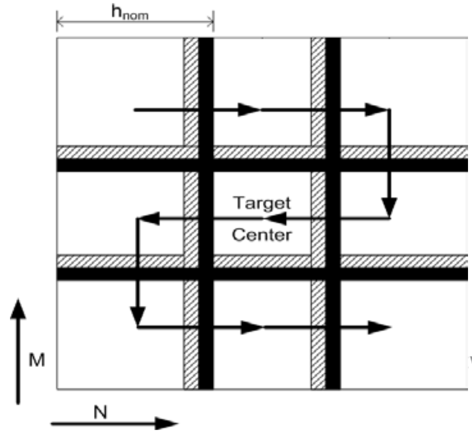
- APE: instantaneous pointing at the right scene
- MPE: average pointing at the right scene
- RPE: stability during payload integration time
- PDE: drift of MPE within one observation period
- PRE: same as PDE, but over different observation periods

t: instantaneous time
ΔT: window time
ΔTs: window stability time







Ch. 5 - Terminology – Pointing Error Indices

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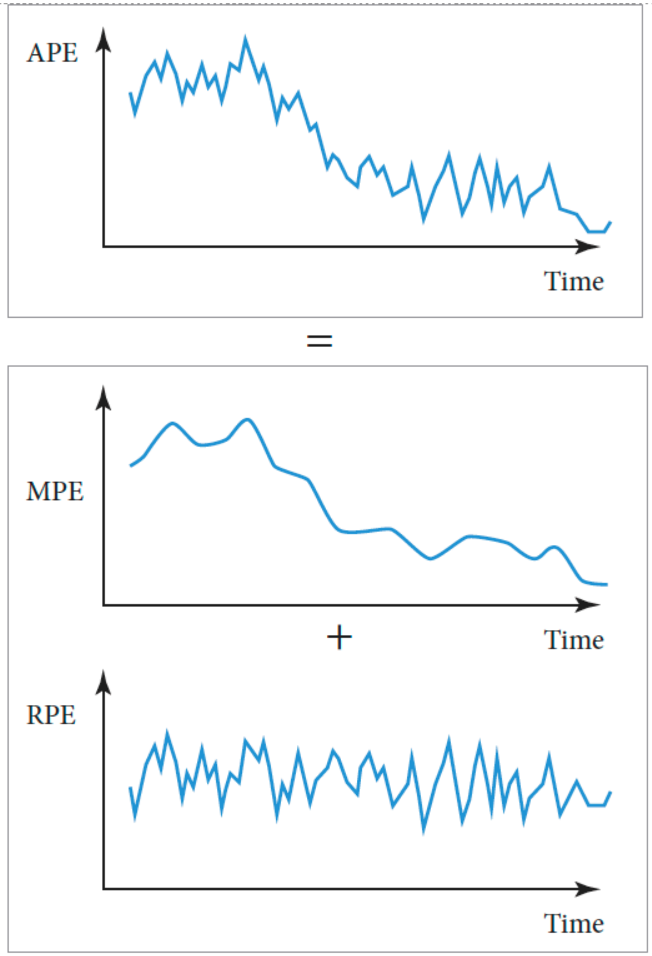
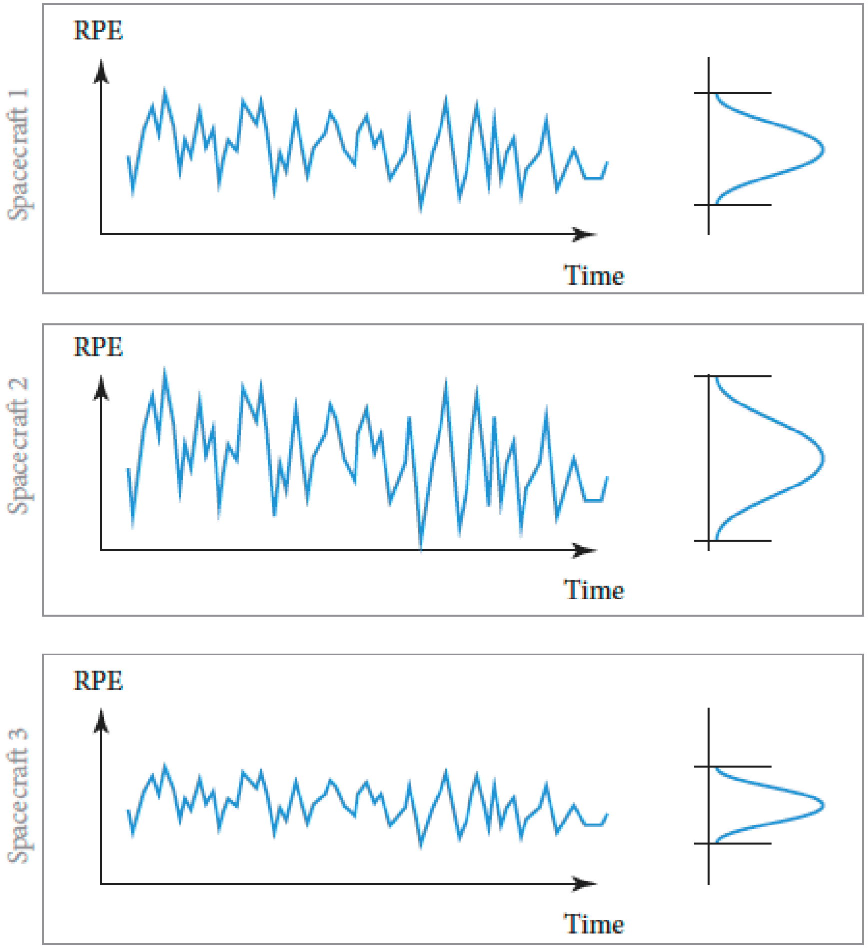
- The MPE represents the **low-frequency** portion of the APE
- The RPE represents the **high-frequency** portion of the APE
- The APE consists of the **combination** of the two: APE = MPE + RPE (+bias)



Ch. 5 - Statistical interpretation



- Ensemble interpretation: worst case in time.
- Temporal interpretation: for the worst case member of the statistical ensemble.
- Mixed interpretation: any time, any member.

Ch. 5 - Statistical interpretation

- The *n-σ* notation shall be restricted to cases where the Gaussian distribution holds. (due to central limit theorem, this hypothesis may be applicable)

- Ensemble interpretation: The error is less than X_{max}, for P_c members of the statistical ensemble, at the worst case in time.
- Temporal interpretation: The error is less than X_{max}, for P_c of the time, for the worst case member of the statistical ensemble.
- Mixed interpretation: The error is less than X_{max}, with probability P_c for a random member of the statistical ensemble, at a random time.

Note: for the temporal interpretation, often the members of the ensemble that are outliers (e.g. outside of *3σ*) are discarded. Similarly, for Gaussian random errors, the ensemble interpretation takes the 3*σ* as the worst-case in time.

→ SPECIFYING A PERFORMANCE REQUIREMENT (SECTION 4.1)

- The following needs to be defined for a complete pointing requirement:
  - Required error value (e.g. 0.1°)
  - Reference frame and axis of angular deviation (or half-cone) (e.g. pitch axis in orbital frame)
  - Error index: APE, MPE, RPE, PDE, PRE (or knowledge error)
  - Statistical interpretation: ensemble, temporal or mixed
  - Confidence level P_c.
  - Evaluation period (e.g. in fine pointing mode)
  - Optional: PSD profile in bandwidth of interest
- This is mathematically expressed as:

$$prob\left(|X| < X_{\max}\right) \geq P_C$$

- Example requirements:
  - *"The instantaneous half cone angle between the actual and desired payload boresight directions shall be less than 1,0 arcmin for 95 % of the time"*
  - *"Over a 10 second integration time, the Euler angles for the transformation between the target and actual payload frames shall have an RPE less than 20 arcsec at 99 % confidence, using the mixed statistical interpretation."*

Using error budgeting to assess compliance (section 4.2)

- A common way to assess compliance is to compile an error budget
  - This involves **taking the known information** about the contributing sources, then **combining them** to estimate the behaviour of the overall performance error, which can then be compared to the original requirement.
  - It is important to emphasise that the common methods of budgeting are **approximate** only, and therefore must be **used with care**. They are based on the assumption from the central limit theorem that the distribution of the total error is Gaussian, and therefore completely specified by its mean and variance per axis.
  - It is not possible to give quantitative limits on its domain of validity: **a degree of engineering judgement is involved**.
- Assessment of correlation between error terms: e.g. correlated by temperature

Using error budgeting to assess compliance (section 4.2) (continued)

- Then contributing errors are classified into groups and characterised
  - A common classification is to distinguish between **biases, random errors, harmonic errors** with various periods, etc.
    - All errors which can potentially be correlated with each other shall be classified in the same group.
    - But a group shall not contain a mixture of **correlated** and **uncorrelated** errors
  - **The period of variation (short term, long term, systematic) is not a sufficient classification criterion**, as by itself it provides no insight into whether or not the errors can be correlated.
  - For each error source, a **mean** and **standard deviation** shall be allocated along each axis. Guidelines for obtaining these parameters are given in **Annex B** and are complemented by the methodology presented in the **Pointing Error Engineering Handbook**.

Ch. 5 - Performance requirements and budgeting

- Combining correlated error sources (collect in groups):

$$\mu_{\text{tot}} = \sum_{i=1}^{N} \mu_i \quad \text{and} \quad \sigma_{\text{tot}} = \sum_{i=1}^{N} \sigma_i$$

- Combining uncorrelated error sources:

$$\mu_{\text{tot}} = \sum_{i=1}^{N} \mu_i \quad \text{and} \quad \sigma_{\text{tot}} = \sqrt{\sum_{i=1}^{N} \sigma_i^2}$$

- Combining different groups:

$$\mu_{\text{tot}} = \sum_{k=1}^{\text{groups}} \mu_k \quad \text{and} \quad \sigma_{\text{tot}} = \sqrt{\sum_{k=1}^{\text{groups}} \sigma_k^2}$$

- *Groups shall not contain both correlate and uncorrelated errors.*

Contents of ECSS-E-HB-60-10A

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- Extension transversally (other subsystems) and vertically (up to system level)
- Control performance engineering tasks during the project phases:
  - Requirements definition
  - Design and configuration
  - Analysis
  - Verification and validation
- Guidelines for extrinsic control performance (pointing performance)
  - Formulation of requirements, selection of indices and interpretations
  - Verification: Monte Carlo vs budgetting
- Guidelines for intrinsic control performance (stability and robustness)
  - Basic control theory, stability margins, sensitivity and complementary sensitivity functions
  - Performance indicators: settling time, overshoot, tracking performance,...
- SPOT-5 example

Conclusion

- The Control Performance Standard E-ST-60-10C provides a set of normative definitions, budget rules, and specification templates needed to manage performance aspects of control systems in the frame of space projects.
- The standard is split into two main clauses:
  - Performance error indices and analysis methods
  - Stability and robustness specification and verification for linear systems
- This normative standard is accompanied by the guidelines provided in ECSS-E-HB-60-10A - Control Performance Guidelines and especially in ESSB-E-HB-003 – Pointing Error Engineering Handbook
- This standard together with the Pointing Error Engineering Handbook provides all the mathematical elements to carry out pointing error budgeting activities
- Exact error budget compilation and calculation can be done in PEET. (assuming linear systems)

ESA Pointing Error Engineering Handbook

ESSB-HB-E-003 Issue 1

The ECSS E60 branch

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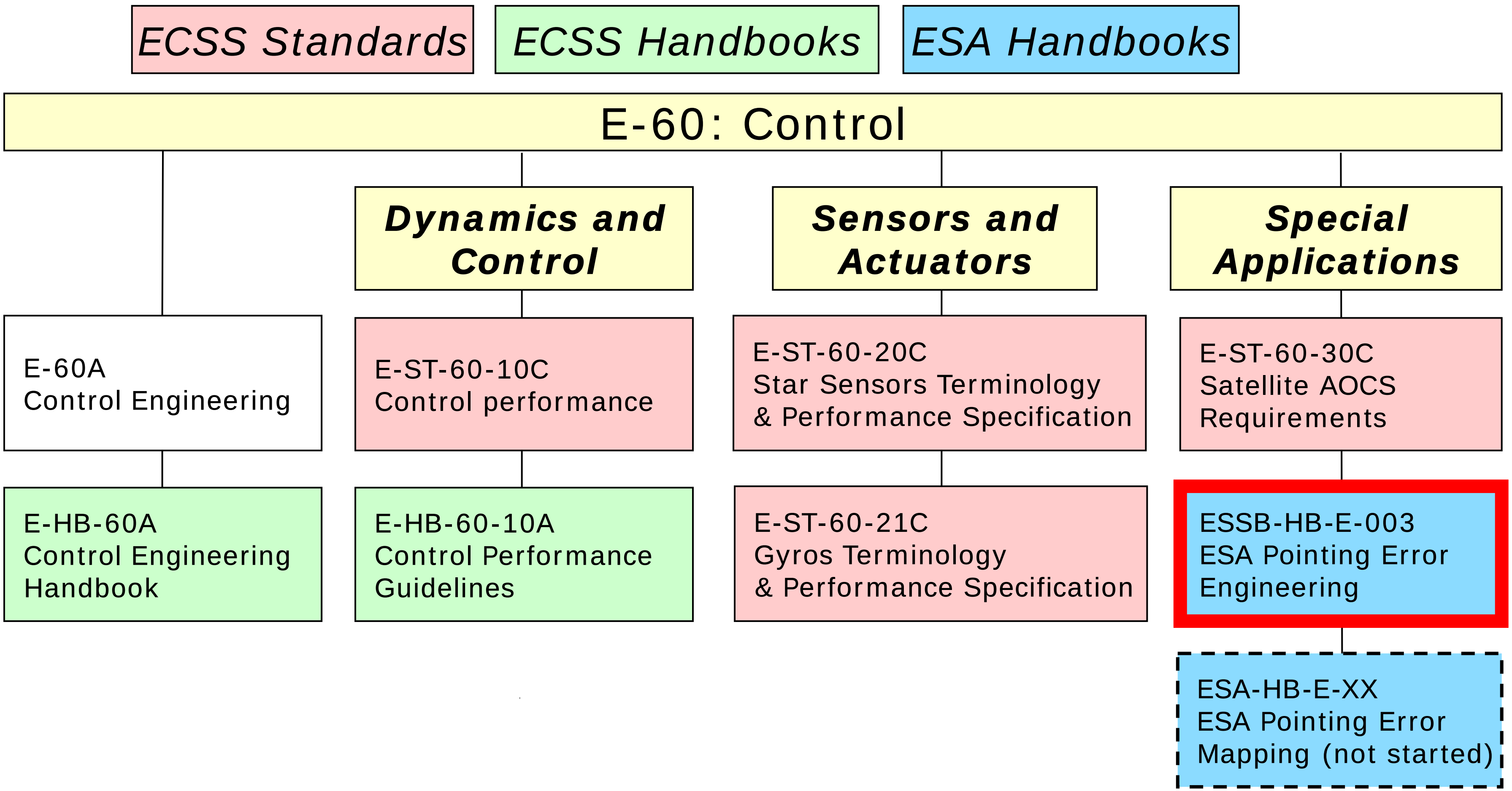
E60 discipline: Control



Introduction

Standardization training program
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- This handbook provides a **consistent and mathematically justified set of guidelines and summation rules** to define an engineering process to build pointing error budgets.
- It embeds the elements of the E-ST-60-10C standard in a **step-by-step engineering process**, providing guidelines for:
  - Characterising pointing error sources
  - Analysing the pointing error source contribution to the ECSS pointing error indexes
  - Compiling system pointing error budgets
- It replaces the old ESA Pointing Error Handbook Ref. ESA-NCR-502 (19 Feb 1993)
- It was adopted in July 2011 in the **LEAS** (List of ESA Applicable Standards)
- A prototype of the accompanying tool PEET (Pointing Error Engineering Tool) has been developed in 2012, now finalized in 2017.

Table of contents

Standardization training program
E60 discipline: Control

ESA Pointing Error Engineering Handbook

Chapter 1,2
Scope & references
- Scope
- ECSS standards
- Other references

Chapter 3
Terms, definitions and abbreviated terms
- Abbreviated terms
- Symbols

Chapter 4
Pointing error: from sources to system performance
- Pointing error sources and contributors
- Time-windowed pointing errors

Chapter 5
Pointing error engineering framework
- Overview
- Methodology
- Framework elements

Chapter 6
Pointing error requirement formulation
- Overview
- Specification parameters
- Notes

Chapter 7-11
Pointing error analysis methodology
- Methodology
- Analysis STep 1
- Analysis STep 2
- Analysis STep 3
- Analysis STep 4

Quick insight

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- Chapter 4 introduces the fundamental definitions
  - Pointing error source (PES) and contributors (PEC), system transfers, time-windowed pointing errors, mathematical formulation of error indexes
- Chapter 5 describes the pointing error engineering framework
  - The methodology of the engineering process for establishing pointing error requirements, their systematic analysis and eventually compliance verification and the related mathematical elements are described
- Chapter 6 provides guidelines for pointing error requirements formulation
  - Includes notes on reference frames, statistical interpretation, evaluation period, level of confidence and requirement break-down and allocation
- Chapters 7-11 define the step-by-step methodology for budget compilation
  - Pointing error source characterisation, pointing error source transfer analysis, error index contribution analysis, system pointing error evaluation
- Annexes support the main chapter and provide complementary information
  - Details on definition of pointing scene, examples on using statistical interpretations, notes on system norms, error metrics and summation rules

Background and Motivation

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- The old ESA Pointing Error Handbook (ESA-NCR-502, 1993) introduced the basic concepts for pointing error budgeting and proposed:
  - an **exact method** for budget compilation (based on PDF characterisation via convolution of different errors, **hardly applicable in practice**)
  - **simplified summation rules** that (can lead to under- or over-estimates of the pointing performance and lack mathematical justification). Examples:
    - Herschel: $\sigma_{total} = \sigma_{Bias} + \sigma_{LongTerm} + \sigma_{ShortTerm}$ (RSS in each class)
    - Sentinel 2: $\sigma_{total} = \sigma_{ST} + \sigma_{B} + \sqrt{(\sigma_H^2 + \sigma_N^2)}$
    - MTG: $\sigma_{total} = \sigma_{ST} + \sqrt{(\sigma_B^2 + \sigma_H^2 + \sigma_N^2)}$
- **Simplified summation rules are not endorsed by E-ST-60-10C**, which foresaw the necessity of a complementary document:
  - “For their own specific purpose, each entity (ESA, national agencies, primes) can further elaborate internal documents, deriving appropriate guidelines and summation rules based on the top level clauses gathered in this E-ST-60-10C standard.”
- Action from ESA Engineering Standardisation Board meeting ESB#40: «To draft and finalize a normative text to replace the ESA Pointing Error HB»

Purpose and approach

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- Purpose of ESA PEEH
  - Pointing Error Budgeting
  - Consistent with and elaborating the E-ST-60-10C ECSS (no additional clause, **no " shall"**)
  - To be used by ESA projects as an **applicable document complementing E-ST-60-10C**
- Approach and added value
  - **Generic process for any mission type and any design phase** through 4 Analysis STeps: AST-1 to -4
  - Added value of frequency domain analysis via PSD on stationary random processes: **windowing in time domain is equivalent to filtering in frequency domain**

AST-1: Pointing Error Sources Characterization

↓

AST-2: Pointing Error Sources Transfer Analysis

↓

AST-3: Error Index Contribution Analysis

↓

AST-4: System Pointing Error Evaluation

Pointing Error Engineering Cycle

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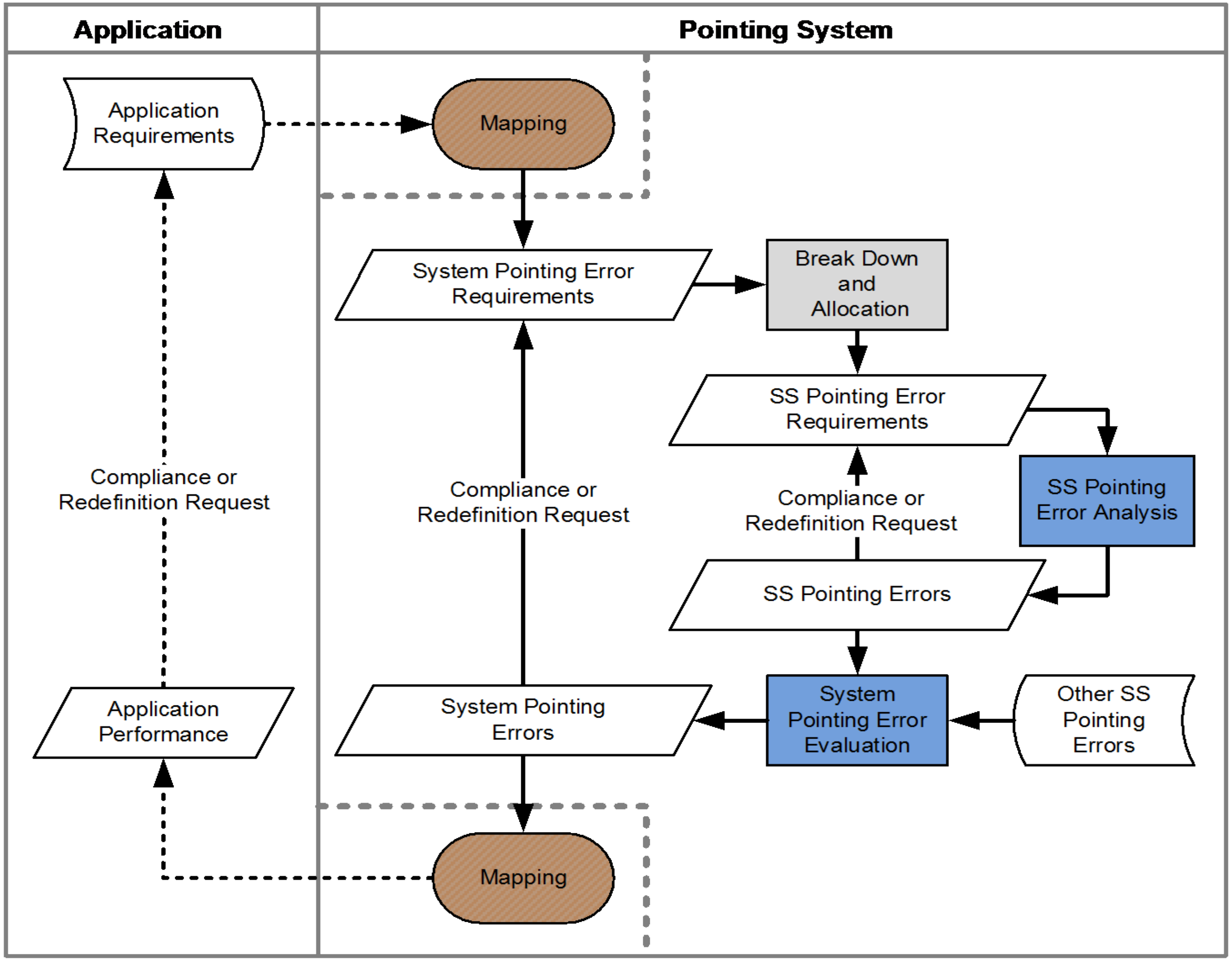
- Mapping from science requirements to spacecraft pointing requirements and identification of pointing error sources (PES) are strongly application specific steps and, as such, are currently excluded from PEEH



Pointing Error Analysis and Evaluation Methodology (1/2)

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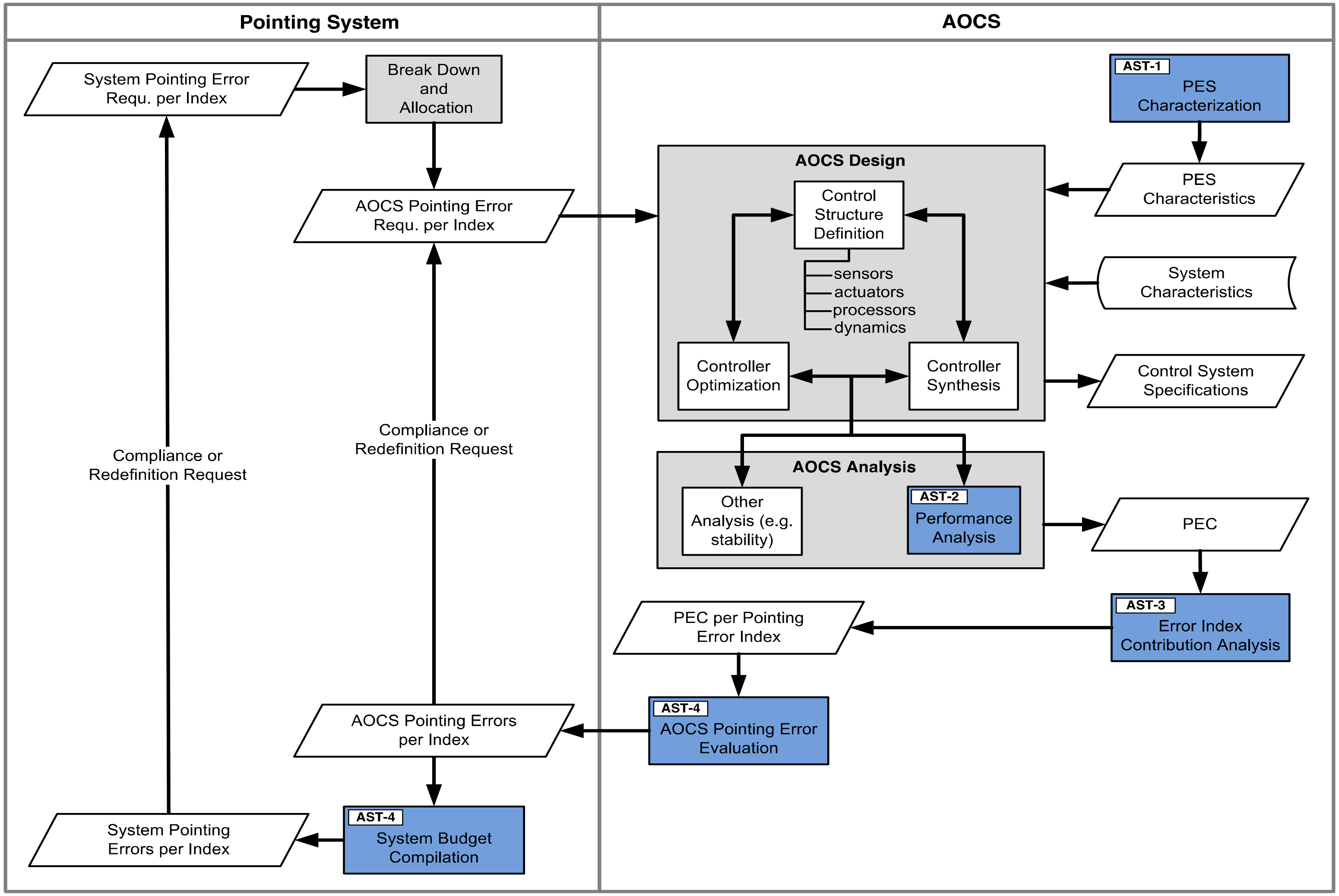

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Pointing Error Analysis and Evaluation Methodology (2/2)

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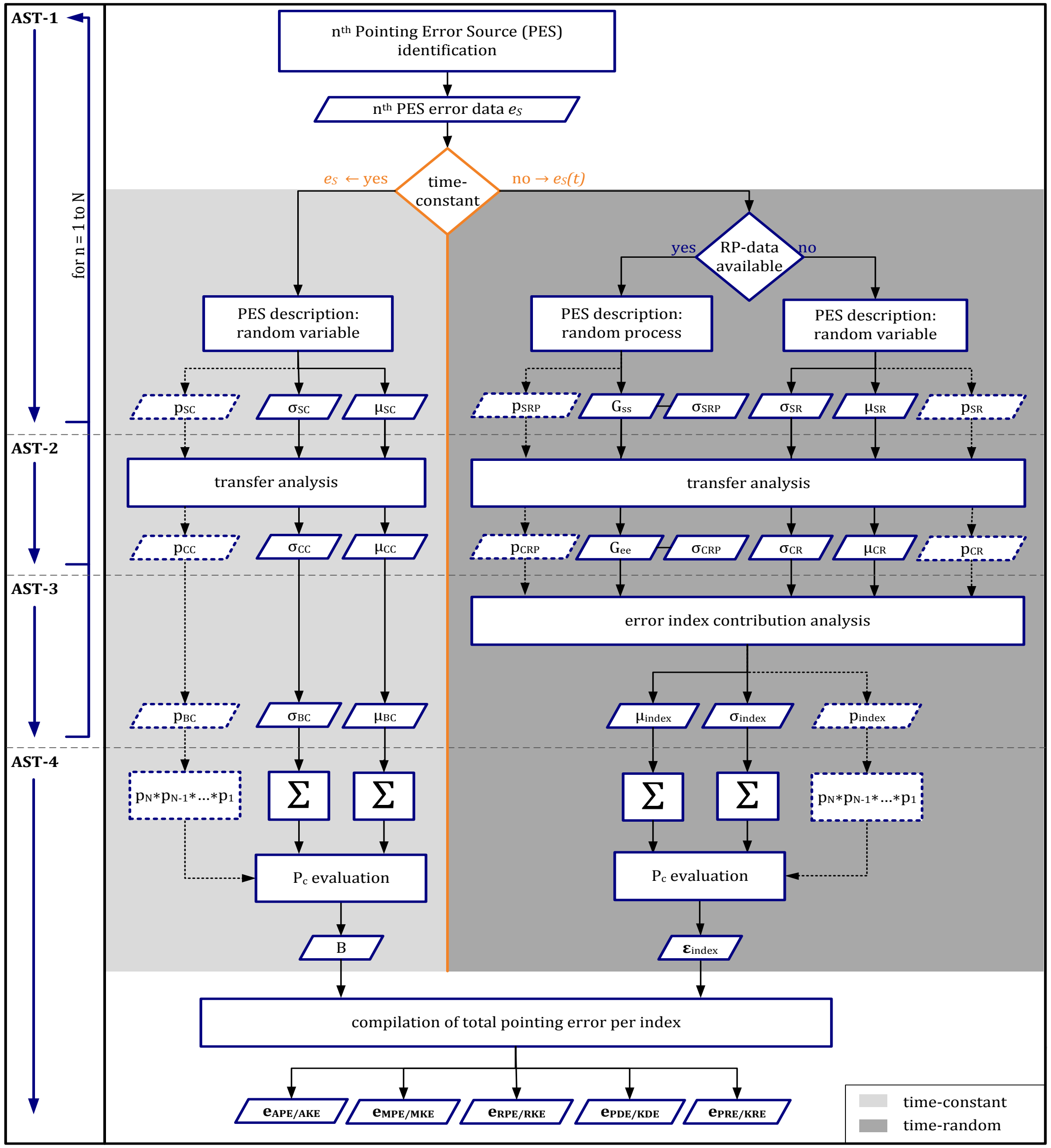
- **Simplified statistical method**: analysis via variance (σ) and mean (μ) and their summation per ECSS error index under the assumption of the central limit theorem, solid lines in figure
- **Advanced statistical method**: analysis by joint PDF characterisation via convolution of different error PDF p…(e), dashed lines in figure



AST-1: PES Characterisation (1/2)

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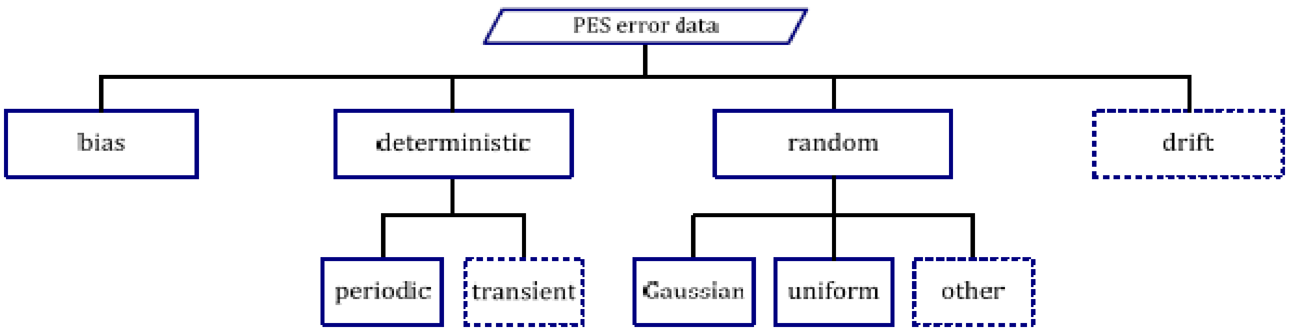
- First step for pointing error sources characterisation is their **categorisation into signal classes** (bias, periodic, random, etc.) that are analogous to those defined and considered in the ECSS standard



- PES data categorisation into signal classes allows simplifying the pointing error analysis **without loss of generality**

AST-1: PES Characterisation (2 / 2)

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- Example: 2 PES with temporal interpretation specified

characterization steps	STR-Payload misalignment error	Gyro-Stellar Estimator (GSE) noise
temporal behavior	time-constant	time-random
signal class	bias	random
description	random variable	random process
reference document	***ECSS***	***ESA PEE Handbook***
characteristic data	$U(0, e_{max})$	$G(0, \sigma_G)$, G_{ee}
temporal interpretation	$p(e)=\delta(e_{max})$	$G(0, \sigma_G)$
AST-1 output data	$\mu(e)=e_{max}$, $\sigma(e)=0$	$\mu(e)=0$, $\sigma(e)=\sigma_G$, G_{ee}

AST-1: PES vs Statistical Interpretation

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- ESA requirement must define the considered statistical population by specifying one of the three **statistical interpretations** introduced in ECSS-E-ST-60-10C:
  - **Ensemble** $\text{Prob}\left[\,|e_{\max}(k)| < e_r\,\right] \geq P_c \quad \text{with} \quad e_{\max}(k) = \max_t\left[\{e_k(t)\}\right]$
    i.e. WC time but error statistically averaged over a population of S/C
  - **Temporal** $\text{Prob}\left[\,|e_{\max}(t)| < e_r\,\right] \geq P_c \quad \text{with} \quad e_{\max}(t) = \max_k\left[\{e_k(t)\}\right]$
    i.e. WC S/C but error statistically averaged over time
  - **Mixed** $\text{Prob}\left[\,|e(k,t)| < e_r\,\right] \geq P_c$
    i.e. time and ensemble considered concurrently (conditional probability)
- The PEEH highlights this **important feature**: the mixed interpretation automatically yields:
  - to the ensemble one if a PES does not randomly vary in time (time constant , e.g. alignment bias)
  - to the temporal interpretation if it does not vary over the population (ensemble-constant, e.g. sensor typical noise).

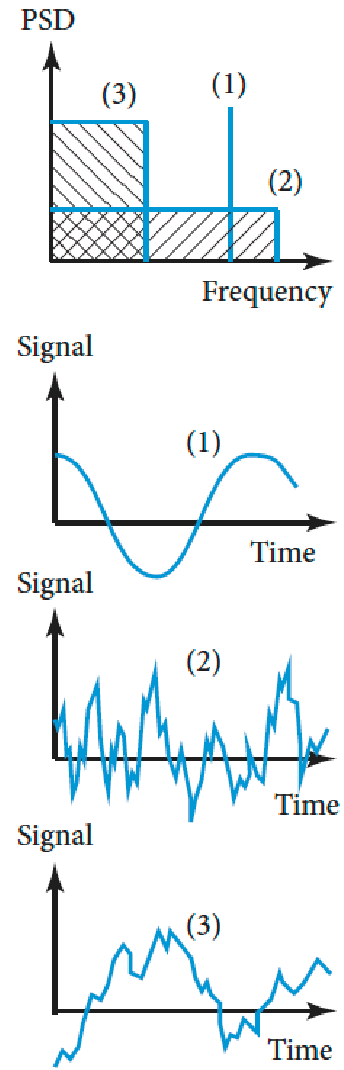
Transfer analysis: Power Spectral Density (PSD) introduction

- Power spectral density (PSD) describes the frequency content of a random process
- The mean square value of the signal is equal to the integral under the PSD curve:

$$E[x^2] = \int_0^\infty S_x(f)df$$

- White noise has a flat PSD curve
- Propagation through a transfer function H:

$$S_y(\omega) = |H(i\omega)|^2 S_x(\omega)$$



AST-2: Transfer Analysis (1/2)

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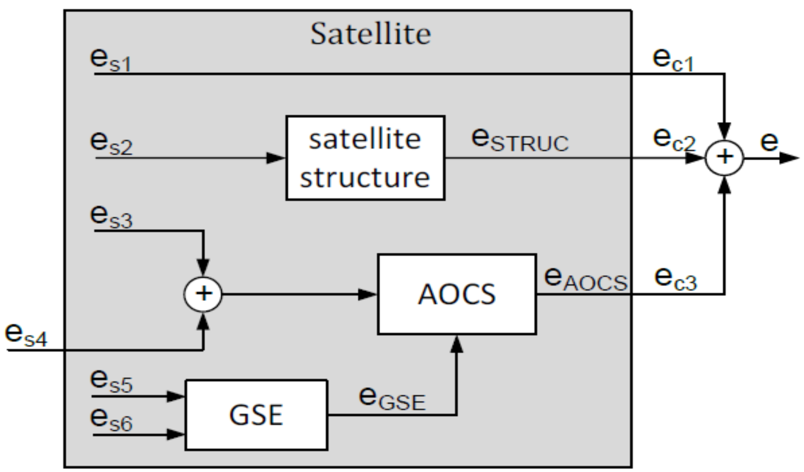
PES (Pointing Error Source) → *transfer* → **PEC** (Pointing Error Contributor)

Transfer from a PES point of origin to the point of interest for error calculation:

- Coordinate frame transformation
- Closed-loop control system
- Thermo-structural transformation



	PES
e_{s1}	STR-Pay. misalignment
e_{s2}	µVibrations
e_{s3}	thruster noise
e_{s4}	solar pressure noise
e_{s5}	*STR noise*
e_{s6}	*gyro noise*

AST-2: Transfer Analysis (2/2)

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- Transfer analysis of time-random PES described as random process can be performed in either time- or frequency-domain:
  - The time-domain approach relies on simulations that, based on the PES PDF or PSD, provide a statistical sample that can be used to characterise the PEC at the point of interest
  - The frequency-domain approach relies on linear transformation of statistical properties
- For LTI systems, the PSD transfer relation can be used:

$$G_{ee}(\omega) = \left|H(j\omega)\right|^2 G_{ss}(\omega)$$

- One **advantage of the frequency-domain approach** is that it can be **used to tune the system transfer function** *H* based on signal and system norms

AST-3: Error Index Contribution Analysis (1/3)

- Time-domain approach is based on simulations and described in the ECSS standard
- Frequency-domain approach provides exact contribution analysis via PSD weighting functions

$$\sigma_e^2 = \frac{1}{2\pi}\int_0^{\infty} G_{ee}(\omega)d\omega$$

$$\Downarrow$$

$$\sigma_{index}^2 = \frac{1}{2\pi}\int_0^{\infty} G_{ee}(\omega)F_{metric}(\omega)d\omega$$

Time-Windowed Error Metrics (Lucke et al. & Pittelkau)

index:= metric		time domain	frequency domain
APE, AKE:= Absolute (ABS) Metric	$\sigma_{index}^2 :=$	$\sigma_{ABS}^2 = E\left[\left(e(t) - \mu_{ABS}\right)^2\right]$	$= \frac{1}{2\pi}\int_0^{\infty} G_{ee}(\omega)d\omega$
MPE, MKE:= Windowed Mean (WM) Metric	$\sigma_{index}^2 :=$	$\sigma_{WM}^2(\Delta t) = E\left[\left(\langle e(t)\rangle_{\Delta t} - \mu_{ABS}\right)^2\right]$	$= \frac{1}{2\pi}\int_0^{\infty} G_{ee}(\omega)\ F_{WM}(\omega,\Delta t)d\omega$
RPE, RKE:= Windowed Variance (WV) Metric	$\sigma_{index}^2 :=$	$\sigma_{WV}^2(\Delta t) = E\left[\left\langle\left(e(t) - \langle e(t)\rangle_{\Delta t}\right)^2\right\rangle_{\Delta t}\right]$	$= \frac{1}{2\pi}\int_0^{\infty} G_{ee}(\omega)\ F_{WV}(\omega,\Delta t)d\omega$
PDE, PRD, KDE, KRE:= Windowed Mean Stability (WMS) Metric	$\sigma_{index}^2 :=$	$\sigma_{WMS}^2(\Delta t,\Delta t_s) = E\left[\left(\langle e(t)\rangle_{\Delta t} - \langle e(t-\Delta t_s)\rangle_{\Delta t}\right)^2\right]$ $\Delta t = \Delta t_1 = \Delta t_2$	$= \frac{1}{2\pi}\int_0^{\infty} G_{ee}(\omega)\ F_{WMS}(\omega,\Delta t,\Delta t_s)d\omega$
PDE, PRD, KDE, KRE:= Stability (STA) Metric	$\sigma_{index}^2 :=$	$\sigma_{STA}^2(\Delta t_s) = E\left[\left(e(t) - e(t-\Delta t_s)\right)^2\right]$	$= \frac{1}{2\pi}\int_0^{\infty} G_{ee}(\omega)\ F_{STA}(\omega,\Delta t_s)d\omega$

AST-3: Error Index Contribution Analysis (2 / 3)

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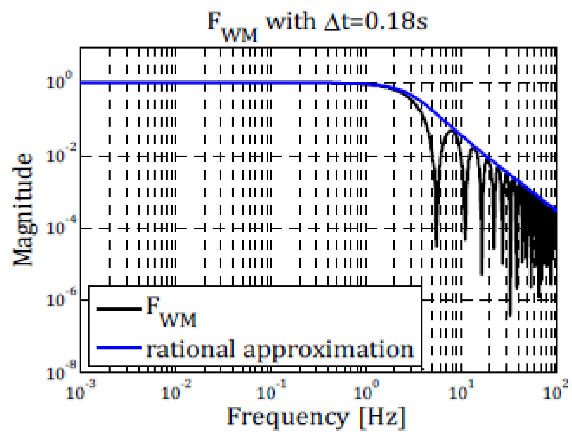
Pointing Error Metric Weighting Functions F_{metric}

Windowed Mean (WM)

$$F_{WM}(\omega, \Delta t) = \frac{2(1-\cos(\omega\Delta t))}{(\omega\Delta t)^2}$$

rational approximation:

$$\tilde{F}_{WM}(s, \Delta t) = \frac{2(s\Delta t + 6)}{(s\Delta t)^2 + 6(s\Delta t) + 12}$$

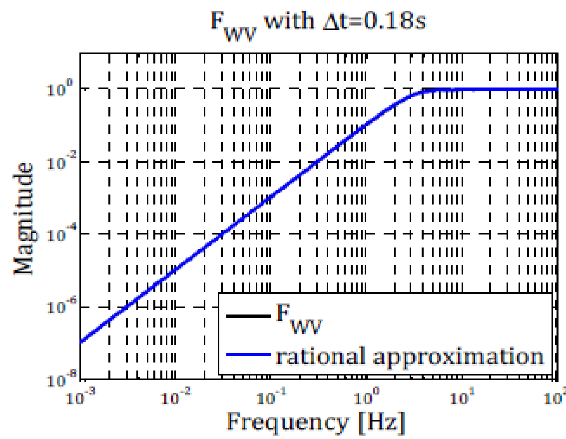


Windowed Variance (WV)

$$F_{WV}(\omega, \Delta t) = 1 - \frac{2(1-\cos(\omega\Delta t))}{(\omega\Delta t)^2}$$

rational approximation:

$$\tilde{F}_{WV}(s, \Delta t) = \frac{s\Delta t\left(s\Delta t + \sqrt{12}\right)}{(s\Delta t)^2 + 6(s\Delta t) + 12}$$

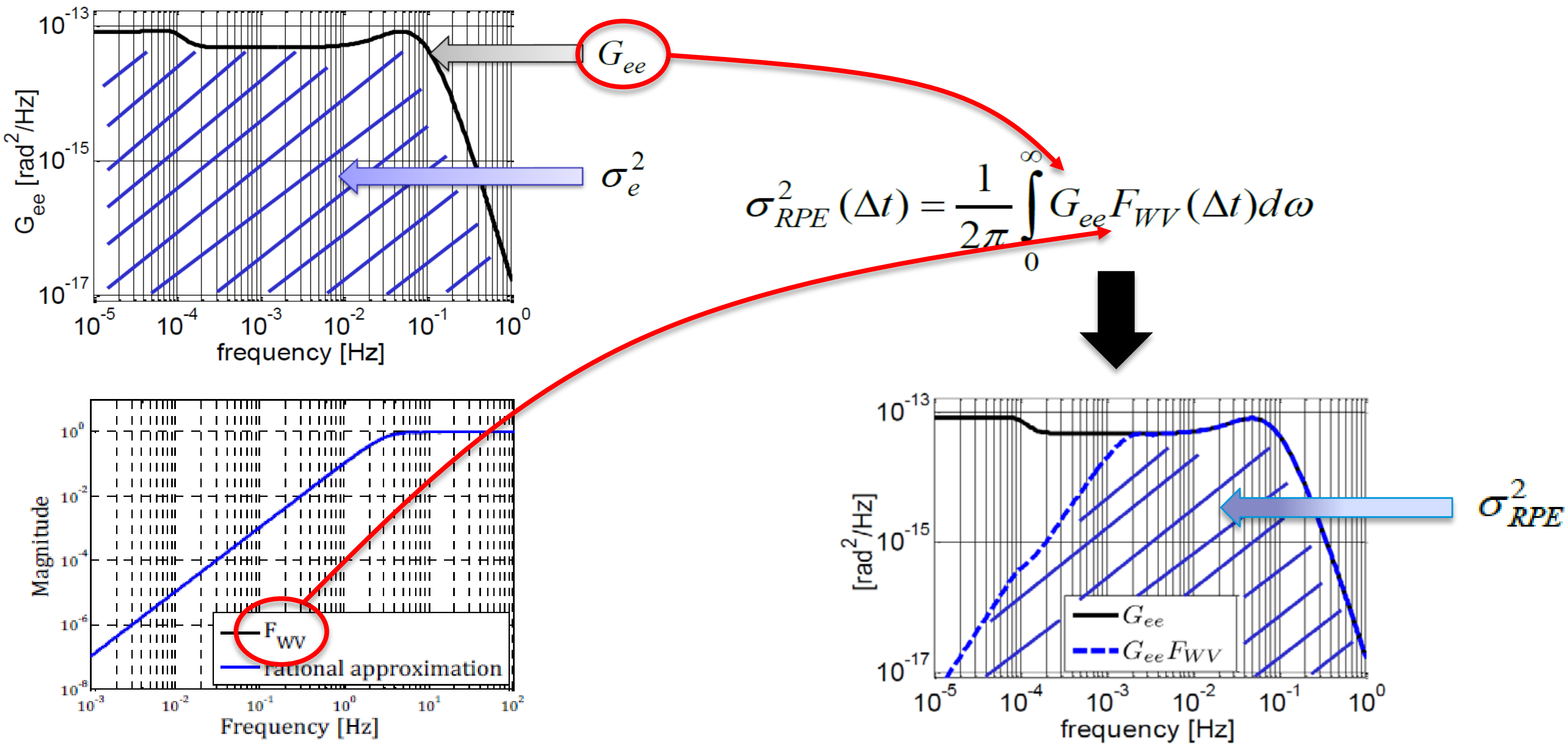


Mean error: MPE, MKE

Relative error: RPE, RKE

AST-3: Error Index Contribution Analysis (3/3)

- Example: GSE noise contribution to RKE



AST-4: Pointing Error Evaluation (1/3)

Standardization training program E60 discipline: Control

- The simplified statistical method is treated in detail in the PEEH
- In the simplified statistical method, time-constant and time-random error contributors (per index) are first summed separately

Time-Constant PEC: Summation and Level of Confidence Evaluation

P_c evaluation	*summation*		
	$\mu_B = \sum_{i=1}^{N} \mu_i$		
$B \le n_p \times \sigma_B + \lvert\mu_B\rvert$	$\sigma_B^2 \le \sigma_c^2 + \sigma_{uc}^2$	*correlated*	$\sigma_c^2 \le N \sum_{i=1}^{N} \sigma_i^2$ (*1) *or* $\sigma_c^2 \le \left[\sum_{i=1}^{N} \sigma_i\right]^2$ (*2)
		uncorrelated	$\sigma_{uc}^2 = \sum_{i=1}^{N} \sigma_i^2$

B	*bias*	P_c	*level of confidence:*
σ^2	*variance*	n_p	*defines max. error complying with P_c requirement of a Gaussian distribution by $n_p \times \sigma$*
μ	*mean*		
N	*number of summands*		

*1: *general worst case upper bound summation (see Annex for derivation).*
*2: *upper bound summation as in ECSS standard.*

AST-4: Pointing Error Evaluation (2 / 3)

- Means are summed linearly
- Uncorrelated variances are RSSed
- Correlated variances summation is estimated by upper bound
- Formulas are valid under the assumption that the central limit theorem applies

<table>
<tr><th colspan="4">TIME-RANDOM PEC: SUMMATION</th></tr>
<tr><th>statistics</th><th colspan="3">summation</th></tr>
<tr><td>μ_{index_sum}</td><td colspan="3">$= \sum_{i=1}^{N} \mu_i$</td></tr>
<tr><td rowspan="4">$\sigma^2_{index\_sum}$</td><td>group summation</td><td colspan="2">signal class groups</td></tr>
<tr><td rowspan="3">$\leq \sum_{i=1}^{N} (\sigma^2_{uc})_i + \sum_{i=1}^{N} (\sigma^2_c)_i$</td><td rowspan="2">correlated</td><td>$\sigma^2_c \leq N \sum_{i=1}^{N} \sigma^2_i$ (*1)
or</td></tr>
<tr><td>$\sigma^2_c \leq \left[\sum_{i=1}^{N} \sigma_i \right]^2$ (*2)</td></tr>
<tr><td>uncorrelated</td><td>$\sigma^2_{uc} = \sum_{i=1}^{N} \sigma^2_i$</td></tr>
<tr><td colspan="4">σ^2 variance N number of summands
μ mean</td></tr>
<tr><td colspan="4">*1: general worst case upper bound summation (see Annex for derivation).
*2: upper bound summation as in ECSS standard □ suggested for periodic error summation.</td></tr>
</table>

AST-4: Pointing Error Evaluation (3 / 3)

Standardization training program E60 discipline: Control

- The applicable confidence level specified in the requirements is then incorporated in the error index evaluation
- Total pointing error per index is finally obtained as sum of time-constant (only for APE, AKE, MPE, MKE) and time-random partial sums

Time-Random PEC: Level of Confidence Evaluation

index	*P_c evaluation assuming applicability of central limit theorem*	*P_c requirement*
APE(Δt_D)/ AKE(Δt_D)	$\varepsilon_{index} = n_p \cdot \sigma_{index,sum}(\Delta t_D) + \lvert \mu_{index} \rvert$	$\text{Prob}(\lvert e \rvert < \varepsilon_{index}) \geq P_c$
MPE(Δt, Δt_D)/ MKE(Δt, Δt_D)	$\varepsilon_{index} = n_p \cdot \sigma_{index,sum}(\Delta t, \Delta t_D) + \lvert \mu_{index} \rvert$	
RPE(Δt, Δt_D)/ RKE(Δt, Δt_D)	$\varepsilon_{index} = n_p \cdot \sigma_{index,sum}(\Delta t, \Delta t_D)$	
PDE(Δt, Δt_s, Δt_D)/ KDE(Δt, Δt_s, Δt_D)	$\varepsilon_{index} = n_p \cdot \sigma_{index,sum}(\Delta t, \Delta t_s, \Delta t_D)$	
PRE(Δt, Δt_s)/ KRE(Δt, Δt_s)	$\varepsilon_{index} = n_p \cdot \sigma_{index,sum}(\Delta t, \Delta t_s)$	

e	*pointing error*	P_c	*level of confidence*
ε_{index}	*max. time-random error compliant with P_c*	n_p	*defines error that complies with P_c requirement of a Gaussian distribution by $n_p \times \sigma_{index}$*
μ_{index}	*mean of time-random error*		
Δt_D	*drift re-set time*		

Conclusions

- The ESA PEEH together with the ECSS standard E-ST-60-10C provides all the mathematical elements to carry out pointing error budgeting activities
- The exact capture of window time and stability properties that is allowed by the frequency domain methodology described by PEEH limits the risk of over- or under-estimates that is inherent in the simplified summation rules
- The frequency domain approach also makes the PEEH methodology a useful tool to complement the tuning activities on system transfer functions (typically attitude estimator, attitude controller, etc.)
- The PEEH methodology can be used to assist requirement breakdown and allocation activities by providing sensitivity evaluation of the pointing performance with respect to the PES
- PEEH has become an [AD] in ESA ITTs in association with the ECSS Control Performance Standard E-ST-60-10C

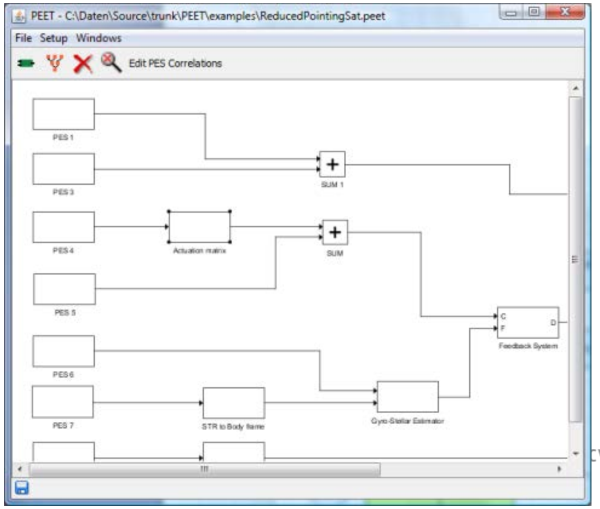
Pointing Error Engineering Tool (PEET)

Standardization training program E60 discipline: Control

- With the aim to complement and to ease the application in daily work of the methodology outlined in the ESA PEEH and the ECSS standard E-ST-60-10C, an accompanying SW tool called PEET has been developed
- Development timeline:
  - 2012 - Prototype of the accompanying SW tool PEET completed (see peet.estec.esa.int)
  - 2013 – Prototype update for positioning and relative pointing
  - 2017 – Fully functional operational version of the accompanying SW tool PEET completed
- Scope of PEET:
  - Translation of the engineering methodology of [ESSB-HB-E-003] into a software tool that supports system engineers
  - General error propagation and analysis tool for the assessment of any type of quantity
  - Systematic compilation of budgets for any mission including high precision pointing missions
  - Common interface for model exchange between departments and users

PEET Software Facts

- Open-source software available free of charge to industry and institutions based in the ESA member states
- Software realized as a Matlab toolbox
  - but no dedicated Matlab knowledge required.
  - analysis algorithms coded in Matlab using well-proven standard functions.
- Dedicated graphical user interface
  - tailored to engineering methodology
  - intuitive Simulink-like system editor with drag & drop functionality for a fast and flexible setup
  - extendable block database with generic and specific models
- Interfaces
  - Data import from Matlab workspace and MS Excel
  - Data export and reporting functionality to MS Excel

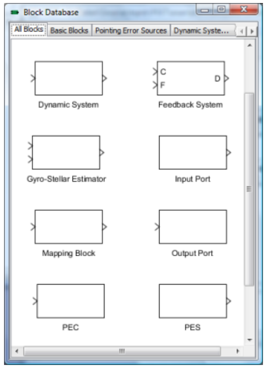


Model based pointing error engineering

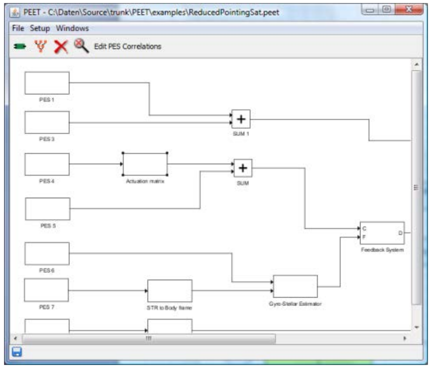
Standardization training program E60 discipline: Control

PEET guides the user in applying the rules defined in [ESSB-HB-E-003] and [ECSS-E-ST-60-10C]. The workflow is fully compatible with the methodology defined in [ESSB-HB-E-003]:

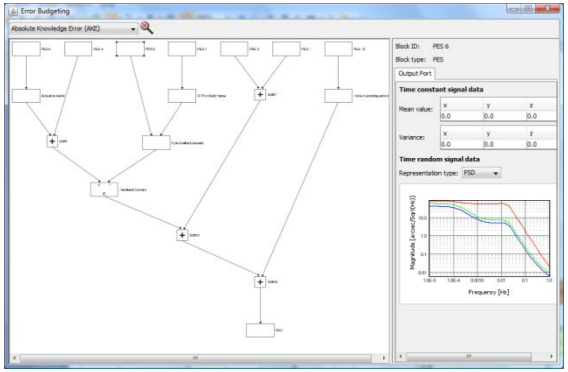
1. Characterize and define pointing error sources using the Block Database (AST-1)
2. Setup the pointing error system within the System Editor of PEET using blocks from the model database (AST-2)
3. Analyse the pointing error budget using the Tree View of PEET (AST-3 & AST-4)



Block Database



System Editor



Tree View

Limitations

- PEET is not a substitute for E2E simulators. It cannot cover:
  - Non-linearities as the transfer analysis is based on LTI systems
  - Transient signals whose properties or transfer behaviour has to be analysed by other means, e.g. time-domain simulations
  - Control design issues as no validity checks are performed concerning the system stability

- The initial classification of error sources cannot be generalized and remains an engineering task.

Sensitivity analyses and system optimization

Standardization training program
E60 discipline: Control

- PEET provides a built-in analysis functionality to assess the sensitivity of error source and system transfer parameters on the final error

- Parameter trade-off studies and system optimization are supported:
  - implicitly, as single parameters (error sources, transfer system,…) and even global settings (statistical interpretation, pointing error index) can easily be modified using the GUI
  - explicitly, as the software provides batch mode capabilities, i.e. the possibility of script-based execution without using the GUI.

Pointing budget break-down

Standardization training program E60 discipline: Control

- Multi-level budgets are inherently supported by proper grouping of subsystems and error evaluations at different “nodes” of the tree.
- An explicit feature allows automatic consistency checks of broken down requirement values and display of requirement violations after the evaluation.