

Standardization training program E-60 discipline: Control

Control Performance Standard

E-ST-60-10C

& ESA Pointing Error Engineering Handbook

ESSB-HB-E-003 Issue 1

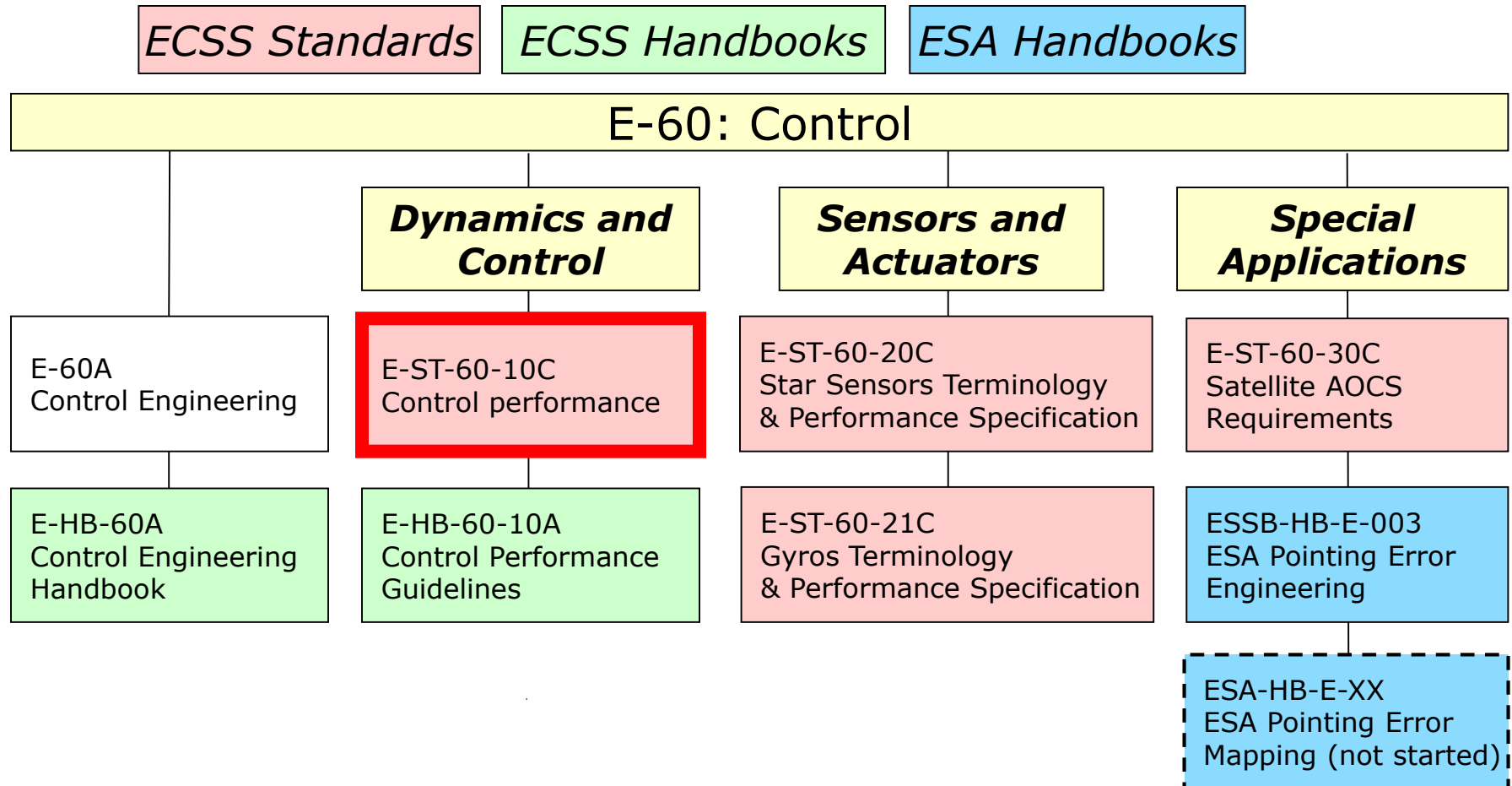
& ESA Pointing Error Engineering Tool (PEET)

Control Performance Standard

E-ST-60-10C

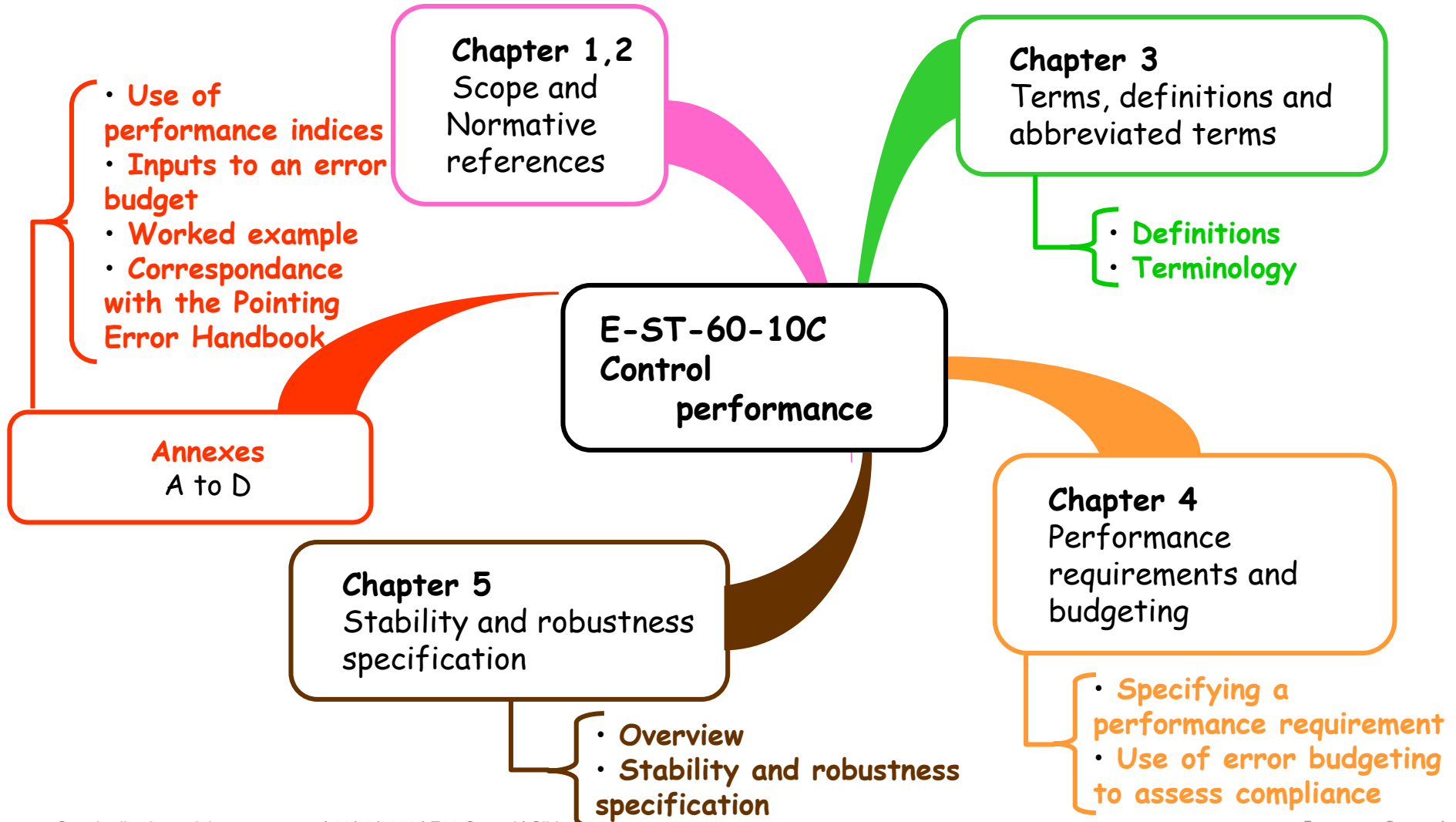
The ECSS E60 branch

Standardization
training program
E60 discipline:
Control



- The standard addresses the issue of control performance, in terms of definition, specification, verification and validation methods and processes.
 - It defines a general framework for handling **performance indicators**, which applies to all disciplines involving control engineering, at different levels ranging **from equipment to system level**.
 - It also focuses on the specific performance indicators applicable to the case of closed-loop control systems – mainly **stability and robustness**.
- Rules are provided for combining different error sources to build a **performance error budget** and use it to assess the **compliance** with a requirement.
 - Complementary material and guidelines for a step-by-step procedure for pointing error budgeting can be found in ECSS-E-HB-60-10A (Control Performance Guidelines) and in ESSB-E-HB-003 (Pointing Error Engineering Handbook)

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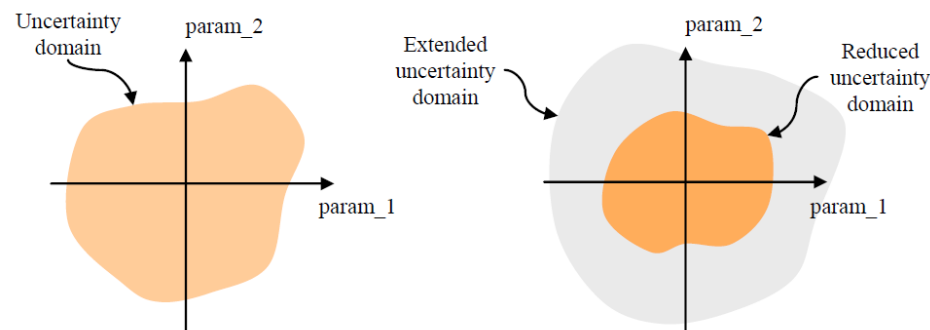
- **Chapter 3** gives the **definition of control performance**-related terms
 - Absolute knowledge error (AKE), absolute performance error (APE), error, error index, individual error source, knowledge error, ...
- **Chapter 4** treats **performance specifications** and rules for **error budgeting**
 - The main features of performance requirements specification are defined together with the fundamental rules for identification, characterisation, and combination of contributors of an error budget.
- **Chapter 5** treats **stability and robustness** specification and verification
 - Stability and robustness concepts for linear systems are introduced and requirements for their specification and verification are defined
- **Annex A** provides **guidelines** on using performance error **indices**
 - Statistical elements are provided that are useful in formulating error requirements and using and building an error budget
- **Annex B** discusses common types of errors and **rules to budget** for them
 - Formulas (some exact, some approximate) are provided to show how typical errors are budgeted for depending on the index and statistical interpretation

Ch. 4 - Stability and robustness specification

- Terminology (section 3)
 - Stability**: ability of a system submitted to bounded external disturbances to remain indefinitely in a bounded domain around an equilibrium position or around an equilibrium trajectory
 - Performance**: the system output meets the accuracy requirements
 - Robustness**: ability of a controlled system to maintain some performance or stability characteristics in the presence of plant, sensors, actuators and/or environmental uncertainties
 - Stability margin**: maximum excursion of some parameters describing a given control system for which the system remains stable
- The state-of-the-art for stability specification is not fully satisfactory.
 - A traditional rule exists, going back to the times of analogue controllers, asking for a gain margin > 6 dB, and a phase margin $> 30^\circ$.
- A more consistent method is proposed for specifying stability and robustness (section 5.2)
 - For SISO loops **the gain margin, the phase margin and the modulus margin** shall be used as default indicators.
 - For MIMO loops use **sensitivity (disturb.) and complementary sensitivity (noise) functions**
 - If other indicators are selected by the supplier, this deviation shall be justified and the relationship with the default ones be established.

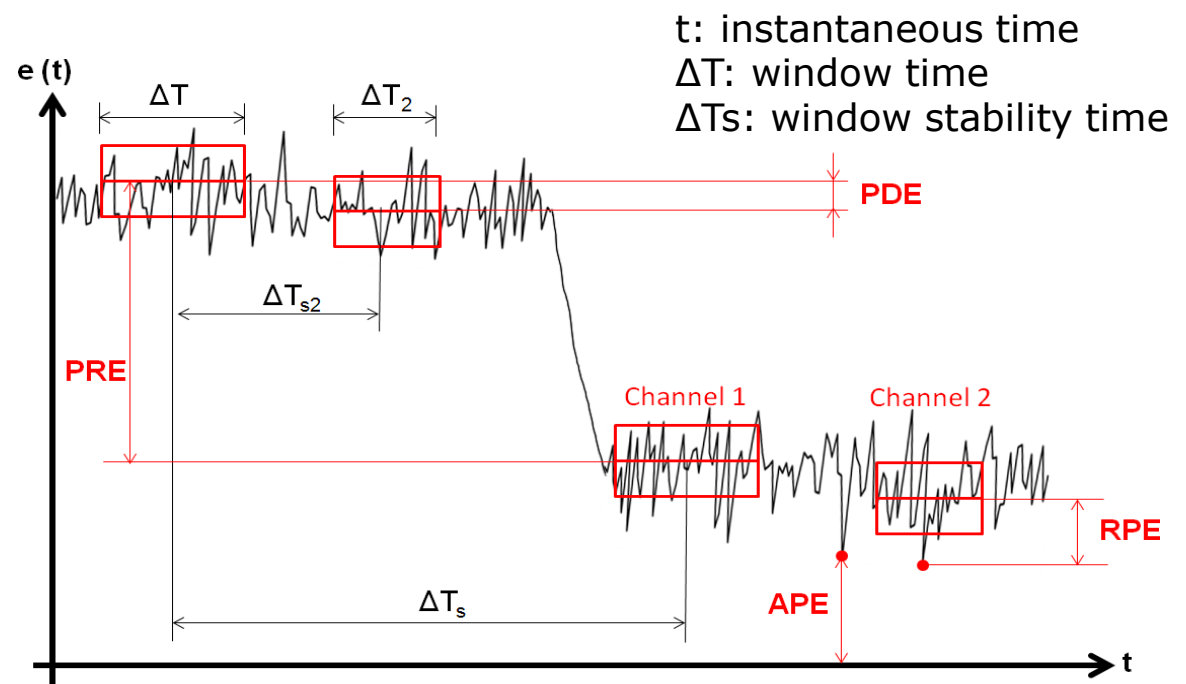
Ch. 4 - Stability and robustness verification

- In order to handle uncertainties (see the previous definition of robustness), **uncertainty domains** must be specified that consist of:
 - A list of **uncertain parameters**
 - For each of these parameters, an interval of uncertainty (or a dispersion) around the nominal value (in percentage or based on a statistical distribution)
 - When relevant, the root cause for the uncertainty (normally lack of characterisation, errors in parameters measurements, **changes over life**)
- A single uncertainty domain or **reduced/extended domains** may be defined
 - Nominal stability margins shall be demonstrated over the entire uncertainty domain (if a single domain is defined – section 5.2.6) or over the reduced domain (if reduced and extended domains are defined - section 5.2.7).
 - Degraded stability margins shall be demonstrated over the extended domain (section 5.2.7).

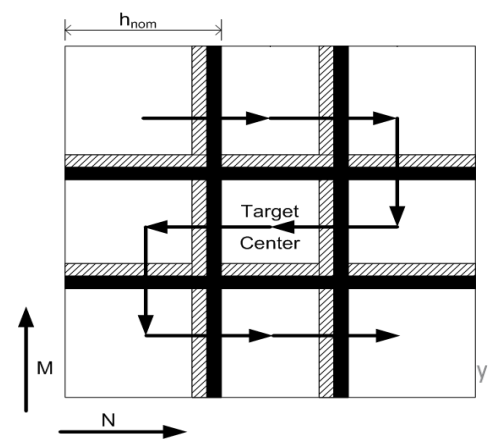
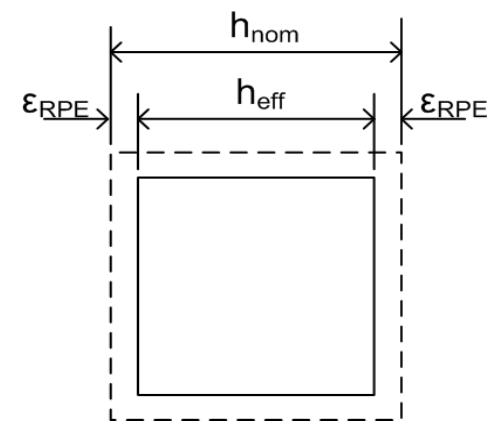


Ch. 5 - Terminology – Pointing Error Indices

- **APE**: instantaneous pointing at the right scene
- **MPE**: average pointing at the right scene
- **RPE**: stability during payload integration time
- **PDE**: drift of MPE within one observation period
- **PRE**: same as PDE, but over different observation periods

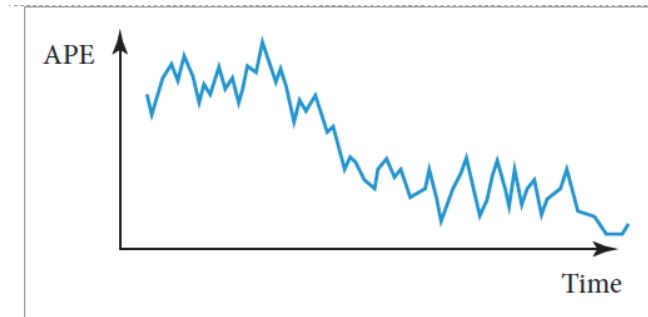


t : instantaneous time
 ΔT : window time
 ΔT_s : window stability time

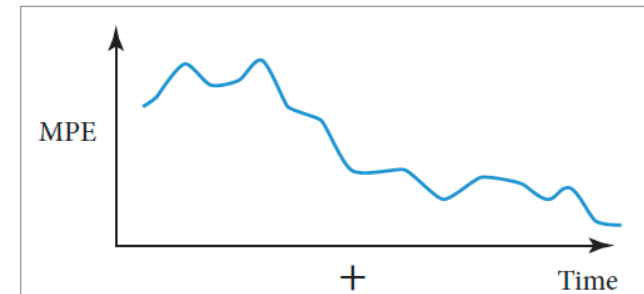


Ch. 5 - Terminology – Pointing Error Indices

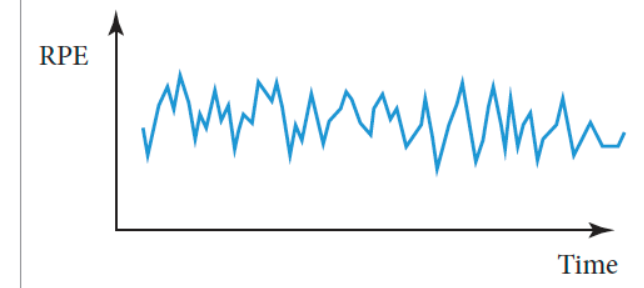
- The **MPE** represents the **low-frequency** portion of the APE
- The **RPE** represents the **high-frequency** portion of the APE
- The **APE** consists of the **combination** of the two: $APE = MPE + RPE (+bias)$



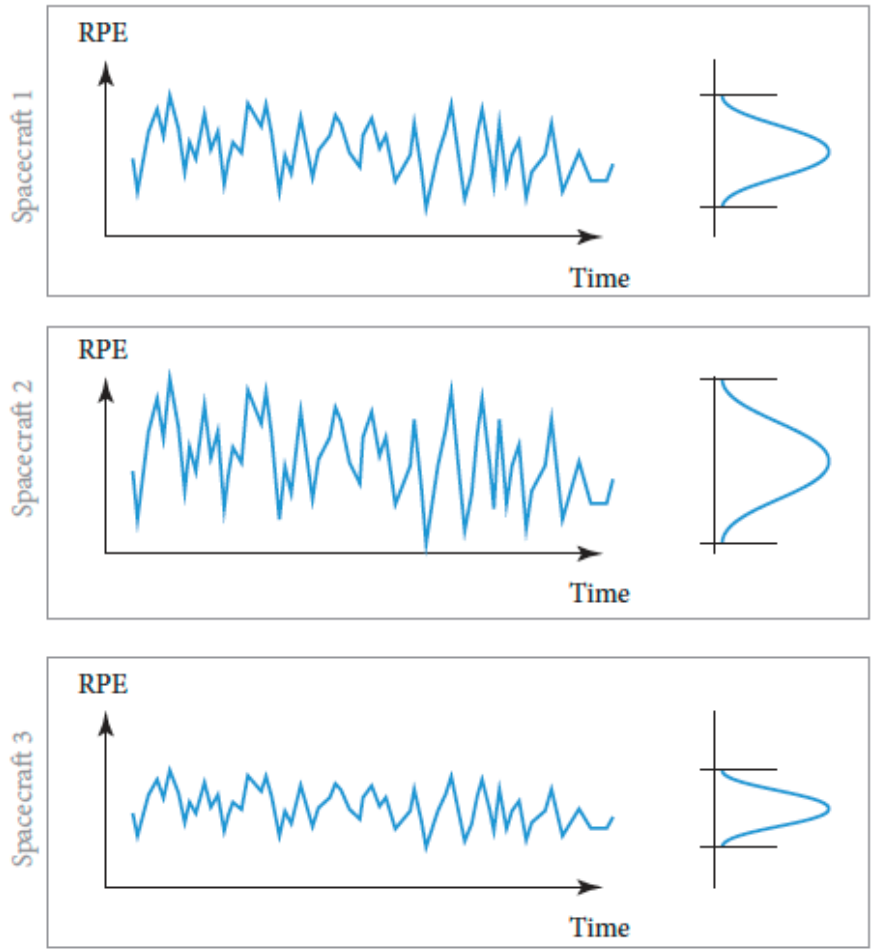
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Ch. 5 - Statistical interpretation



- Ensemble interpretation: worst case in time.
- Temporal interpretation: for the worst case member of the statistical ensemble.
- Mixed interpretation: any time, any member.

- The $n\text{-}\sigma$ notation shall be restricted to cases where the Gaussian distribution holds. (due to central limit theorem, this hypothesis may be applicable)
- **Ensemble interpretation**: The error is less than X_{\max} , for P_c members of the statistical ensemble, at the worst case in time.
- **Temporal interpretation**: The error is less than X_{\max} , for P_c of the time, for the worst case member of the statistical ensemble.
- **Mixed interpretation**: The error is less than X_{\max} , with probability P_c for a random member of the statistical ensemble, at a random time.

Ch. 5 - Performance requirements and budgeting

→ SPECIFYING A PERFORMANCE REQUIREMENT (SECTION 4.1)

- The following needs to be defined for a complete pointing requirement:
 - Required error value (e.g. 0.1°)
 - Reference frame and axis of angular deviation (or half-cone) (e.g. pitch axis in orbital frame)
 - Error index: APE, MPE, RPE, PDE, PRE (or knowledge error)
 - Statistical interpretation: ensemble, temporal or mixed
 - Confidence level P_c .
 - Evaluation period (e.g. in fine pointing mode)
 - Optional: PSD profile in bandwidth of interest

- This is mathematically expressed as:

$$\text{prob}(|X| < X_{\max}) \geq P_c$$

- Example requirements:
 - *"The instantaneous half cone angle between the actual and desired payload boresight directions shall be less than 1,0 arcmin for 95 % of the time"*
 - *"Over a 10 second integration time, the Euler angles for the transformation between the target and actual payload frames shall have an RPE less than 20 arcsec at 99 % confidence, using the mixed statistical interpretation."*

Using error budgeting to assess compliance (section 4.2)

- A common way to assess compliance is to compile an **error budget**
 - This involves **taking the known information** about the contributing sources, then **combining them** to estimate the behaviour of the overall performance error, which can then be compared to the original requirement.
 - It is important to emphasise that the common methods of budgeting are **approximate** only, and therefore must be **used with care**. They are based on the assumption from the central limit theorem that the distribution of the total error is Gaussian, and therefore completely specified by its mean and variance per axis.
 - It is not possible to give quantitative limits on its domain of validity: **a degree of engineering judgement is involved**.
- Assessment of correlation between error terms: e.g. correlated by temperature

Using error budgeting to assess compliance (section 4.2) (continued)

- Then contributing errors are classified into groups and characterised
 - A common classification is to distinguish between **biases, random errors, harmonic errors** with various periods, etc.
 - All errors which can potentially be correlated with each other shall be classified in the same group.
 - But a group shall not contain a mixture of **correlated** and **uncorrelated** errors
 - **The period of variation (short term, long term, systematic) is not a sufficient classification criterion**, as by itself it provides no insight into whether or not the errors can be correlated.
 - For each error source, a **mean** and **standard deviation** shall be allocated along each axis. Guidelines for obtaining these parameters are given in **Annex B** and are complemented by the methodology presented in the **Pointing Error Engineering Handbook**.

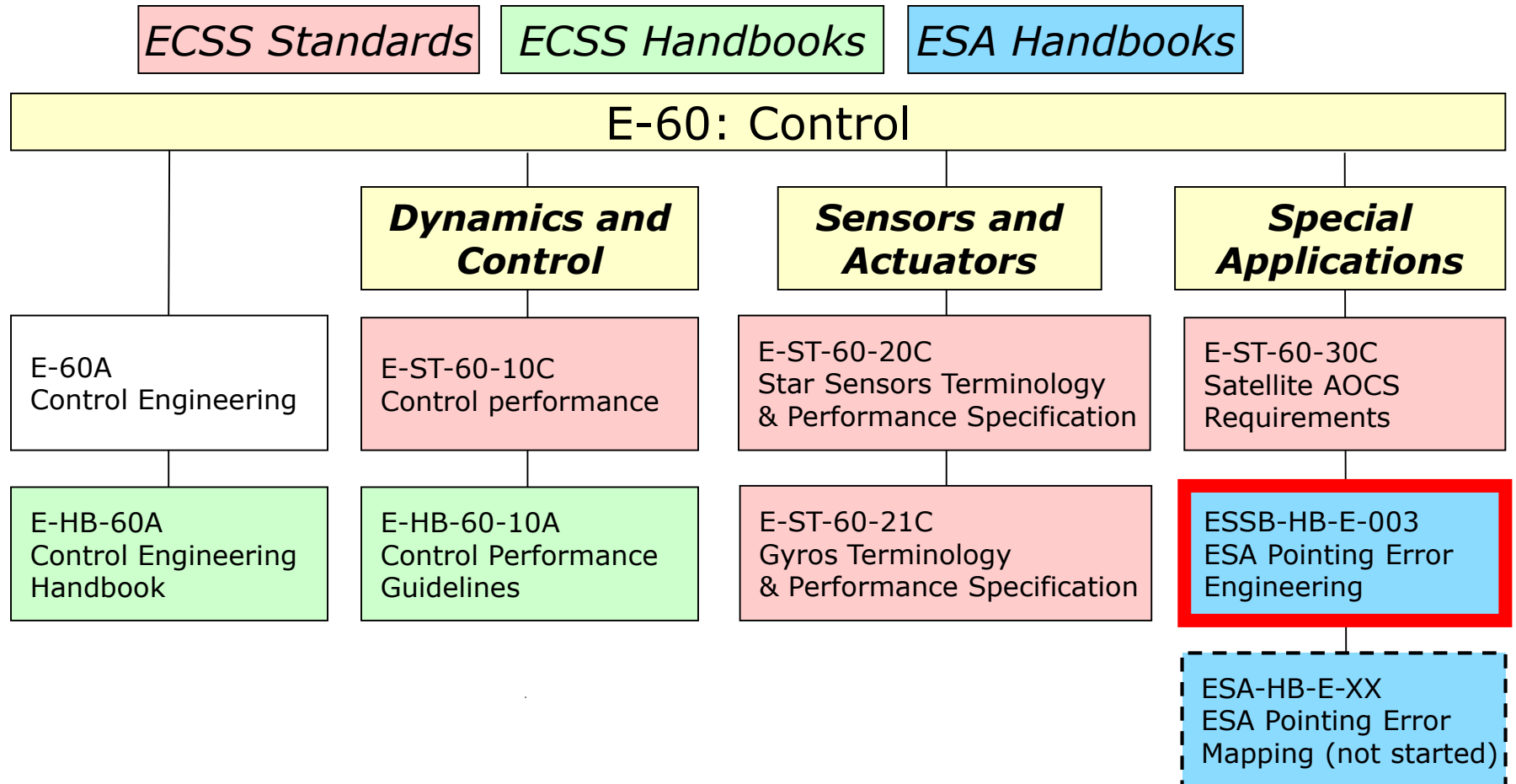
- **Extension** transversally (other subsystems) and vertically (up to system level)
- Control performance engineering **tasks** during the project phases:
 - Requirements definition
 - Design and configuration
 - Analysis
 - Verification and validation
- Guidelines for **extrinsic** control performance (pointing performance)
 - Formulation of requirements, selection of indices and interpretations
 - Verification: Monte Carlo vs budgeting
- Guidelines for **intrinsic** control performance (stability and robustness)
 - Basic control theory, stability margins, sensitivity and complementary sensitivity functions
 - Performance indicators: settling time, overshoot, tracking performance,...
- SPOT-5 **example**

- The Control Performance Standard E-ST-60-10C provides a set of normative definitions, budget rules, and specification templates needed to manage performance aspects of control systems in the frame of space projects.
- The standard is split into two main clauses:
 - Performance error indices and analysis methods
 - Stability and robustness specification and verification for linear systems
- This normative standard is accompanied by the guidelines provided in ECSS-E-HB-60-10A - Control Performance Guidelines and especially in ESSB-E-HB-003 – Pointing Error Engineering Handbook
- This standard together with the Pointing Error Engineering Handbook provides all the mathematical elements to carry out pointing error budgeting activities

***ESA Pointing Error Engineering
Handbook
ESSB-HB-E-003 Issue 1***

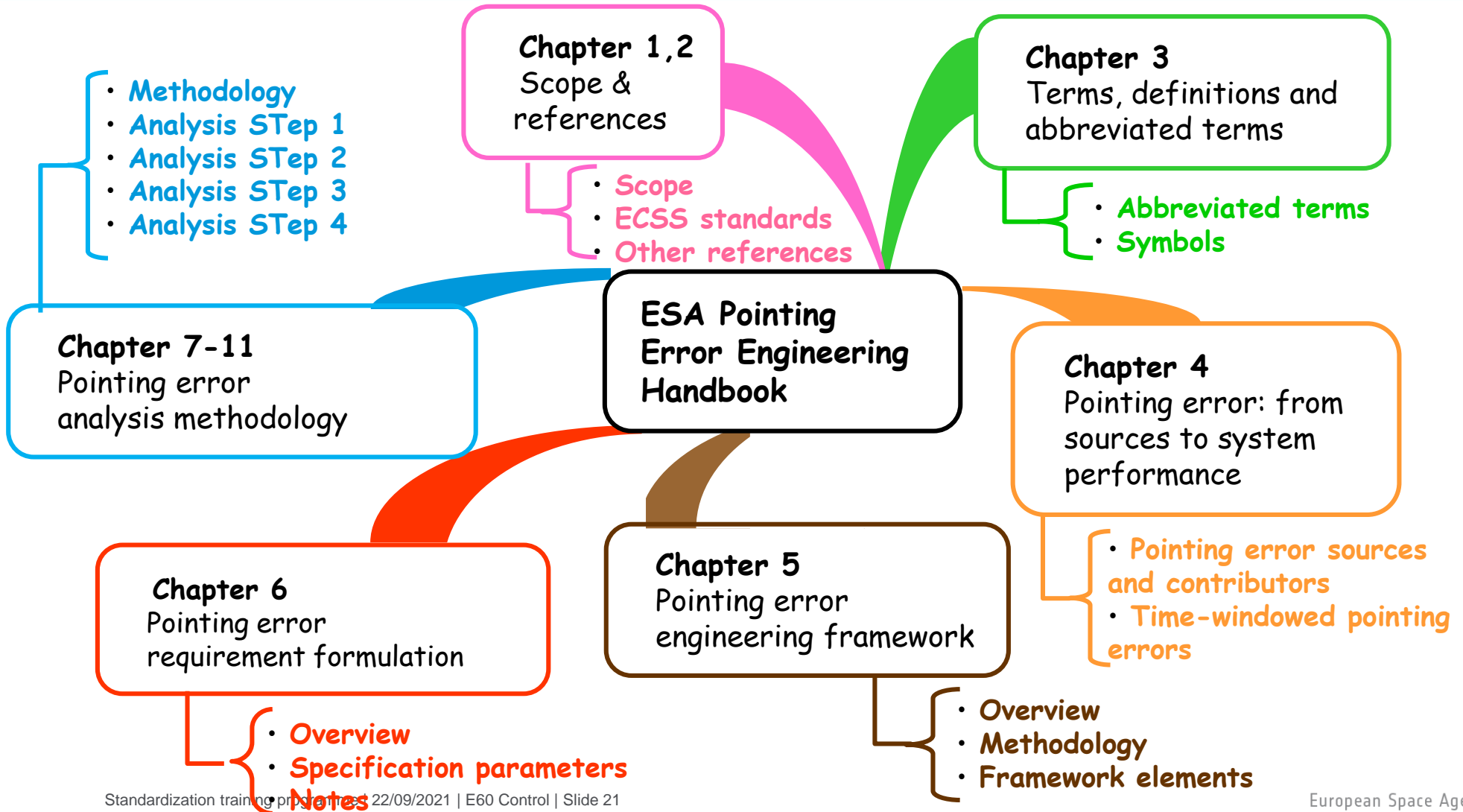
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- This handbook provides a **consistent and mathematically justified set of guidelines and summation rules** to define an engineering process to build pointing error budgets.
- It embeds the elements of the E-ST-60-10C standard in a **step-by-step engineering process**, providing guidelines for:
 - Characterising pointing error **sources**
 - Analysing the pointing error source **contribution** to the ECSS pointing error indexes
 - Compiling system pointing **error budgets**
- It replaces the old ESA Pointing Error Handbook Ref. ESA-NCR-502 (19 Feb 1993)
- It was adopted in July 2011 in the **LEAS** (List of ESA Applicable Standards)
- A prototype of the accompanying tool PEET (Pointing Error Engineering Tool) has been developed in 2012, now finalized in 2017.

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- **Chapter 4** introduces the **fundamental definitions**
 - Pointing error source (PES) and contributors (PEC), system transfers, time-windowed pointing errors, mathematical formulation of error indexes
- **Chapter 5** describes the pointing error engineering **framework**
 - The methodology of the engineering process for establishing pointing error requirements, their systematic analysis and eventually compliance verification and the related mathematical elements are described
- **Chapter 6** provides guidelines for pointing error **requirements formulation**
 - Includes notes on reference frames, statistical interpretation, evaluation period, level of confidence and requirement break-down and allocation
- **Chapters 7-11** define the **step-by-step methodology** for budget compilation
 - Pointing error source characterisation, pointing error source transfer analysis, error index contribution analysis, system pointing error evaluation
- **Annexes** support the main chapter and provide complementary information
 - Details on definition of pointing scene, examples on using statistical interpretations, notes on system norms, error metrics and summation rules

- The old ESA Pointing Error Handbook (ESA-NCR-502, 1993) introduced the basic concepts for pointing error budgeting and proposed:
 - an **exact method** for budget compilation (based on PDF characterisation via convolution of different errors, **hardly applicable in practice**)
 - **simplified summation rules** that (can lead to under- or over-estimates of the pointing performance and lack mathematical justification). Examples:
 - Herschel: $\sigma_{total} = \sigma_{Bias} + \sigma_{LongTerm} + \sigma_{ShortTerm}$ (RSS in each class)
 - Sentinel 2: $\sigma_{total} = \sigma_{ST} + \sigma_B + \sqrt{(\sigma_H^2 + \sigma_N^2)}$
 - MTG: $\sigma_{total} = \sigma_{ST} + \sqrt{(\sigma_B^2 + \sigma_H^2 + \sigma_N^2)}$
- **Simplified summation rules are not endorsed by E-ST-60-10C**, which foresaw the necessity of a complementary document:
 - “For their own specific purpose, each entity (ESA, national agencies, primes) can further elaborate internal documents, deriving appropriate guidelines and summation rules based on the top level clauses gathered in this E-ST-60-10C standard.”
- Action from ESA Engineering Standardisation Board meeting ESB#40:
«To draft and finalize a normative text to replace the ESA Pointing Error HB»

Purpose and approach

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- Purpose of ESA PEEH
 - Pointing Error Budgeting
 - Consistent with and elaborating the E-ST-60-10C ECSS (no additional clause, **no "shall"**)
 - To be used by ESA projects as an **applicable document complementing E-ST-60-10C**
- Approach and added value
 - **Generic process for any mission type and any design phase** through 4 Analysis Steps: AST-1 to -4
 - Added value of frequency domain analysis via PSD on stationary random processes: **windowing in time domain is equivalent to filtering in frequency domain**

AST-1: Pointing Error Sources
Characterization



AST-2: Pointing Error Sources
Transfer Analysis



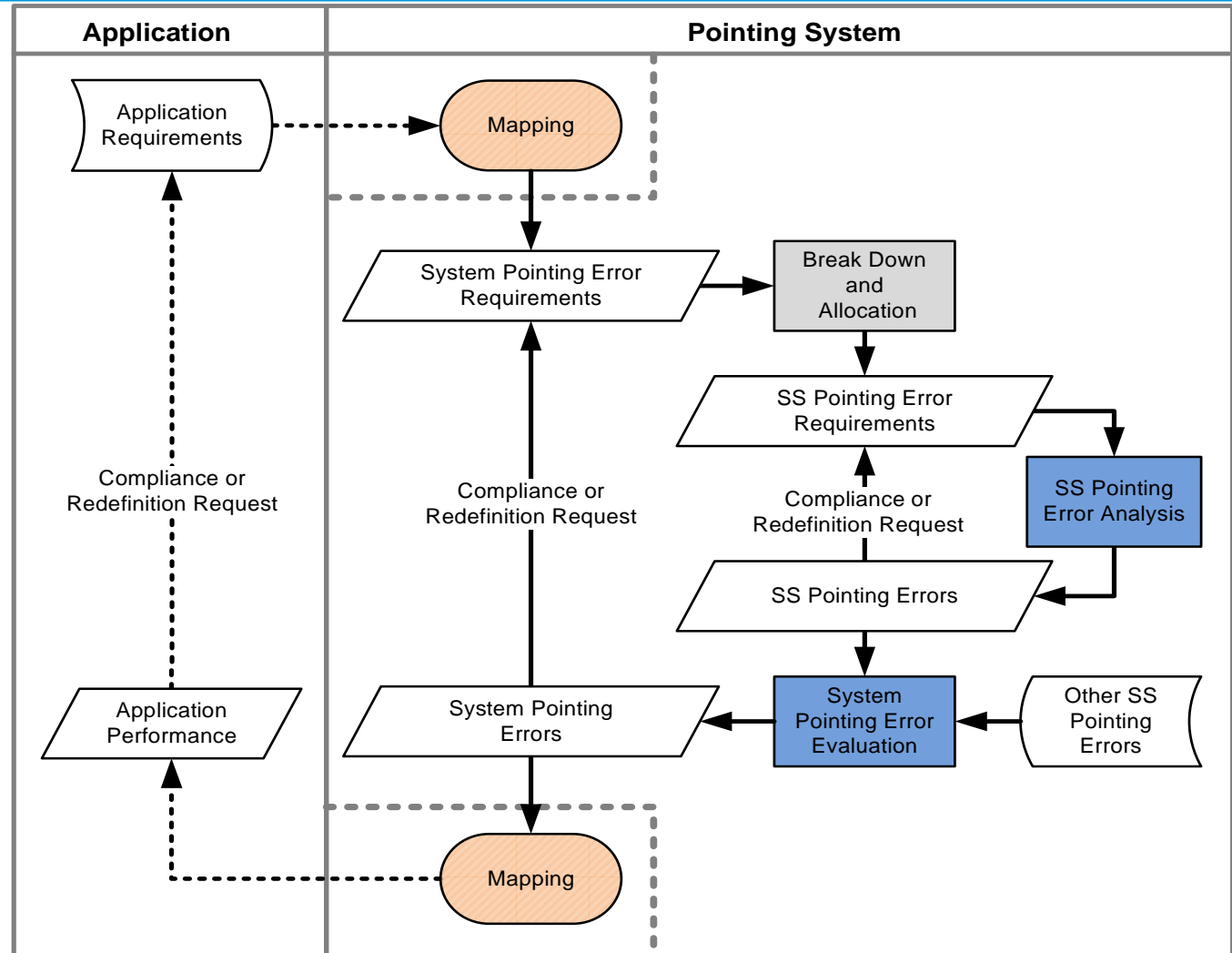
AST-3: Error Index
Contribution Analysis



AST-4: System Pointing Error
Evaluation

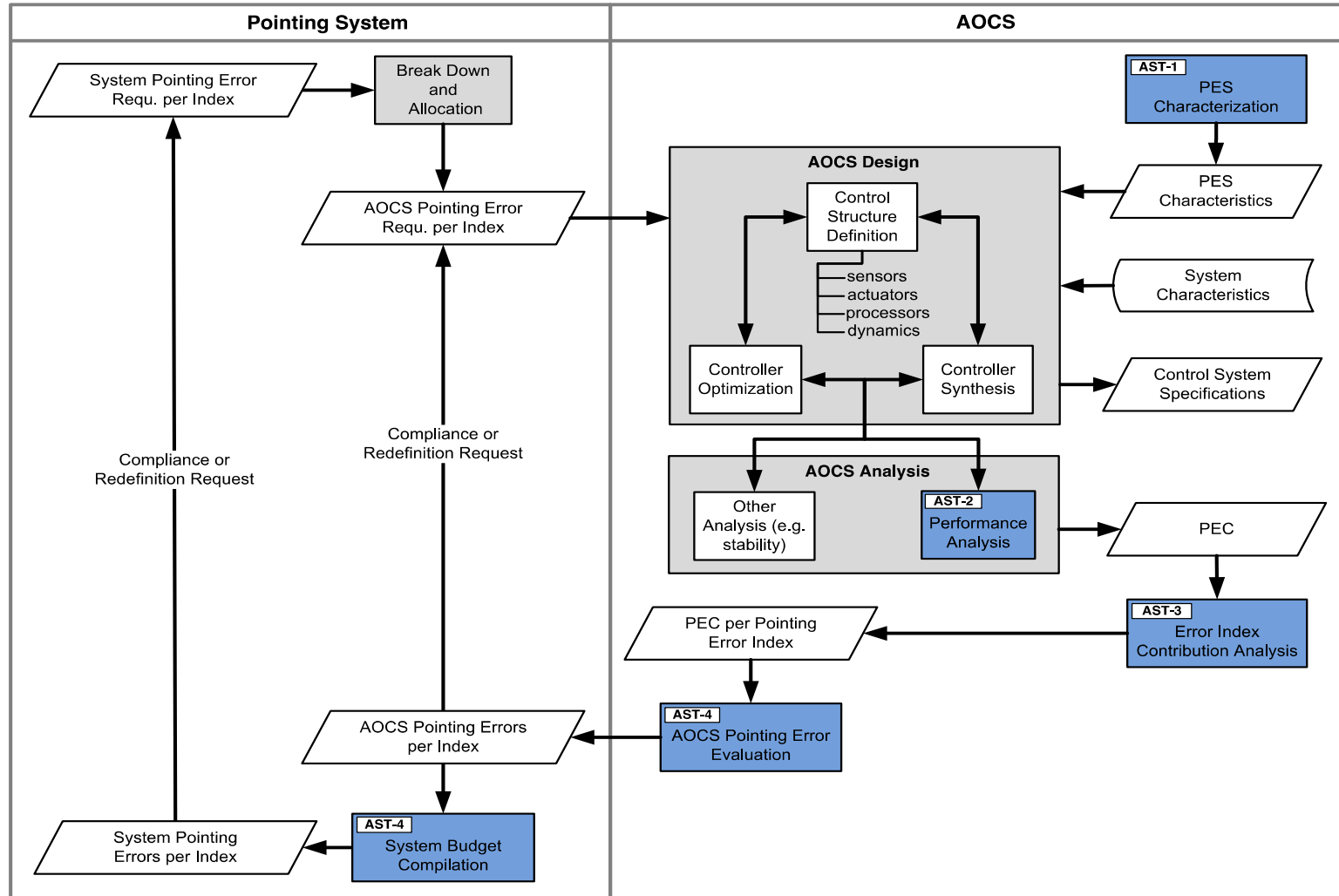
Pointing Error Engineering Cycle

- Mapping from science requirements to spacecraft pointing requirements and identification of pointing error sources (PES) are strongly application specific steps and, as such, are currently excluded from PEEH



Pointing Error Analysis and Evaluation Methodology (1/2)

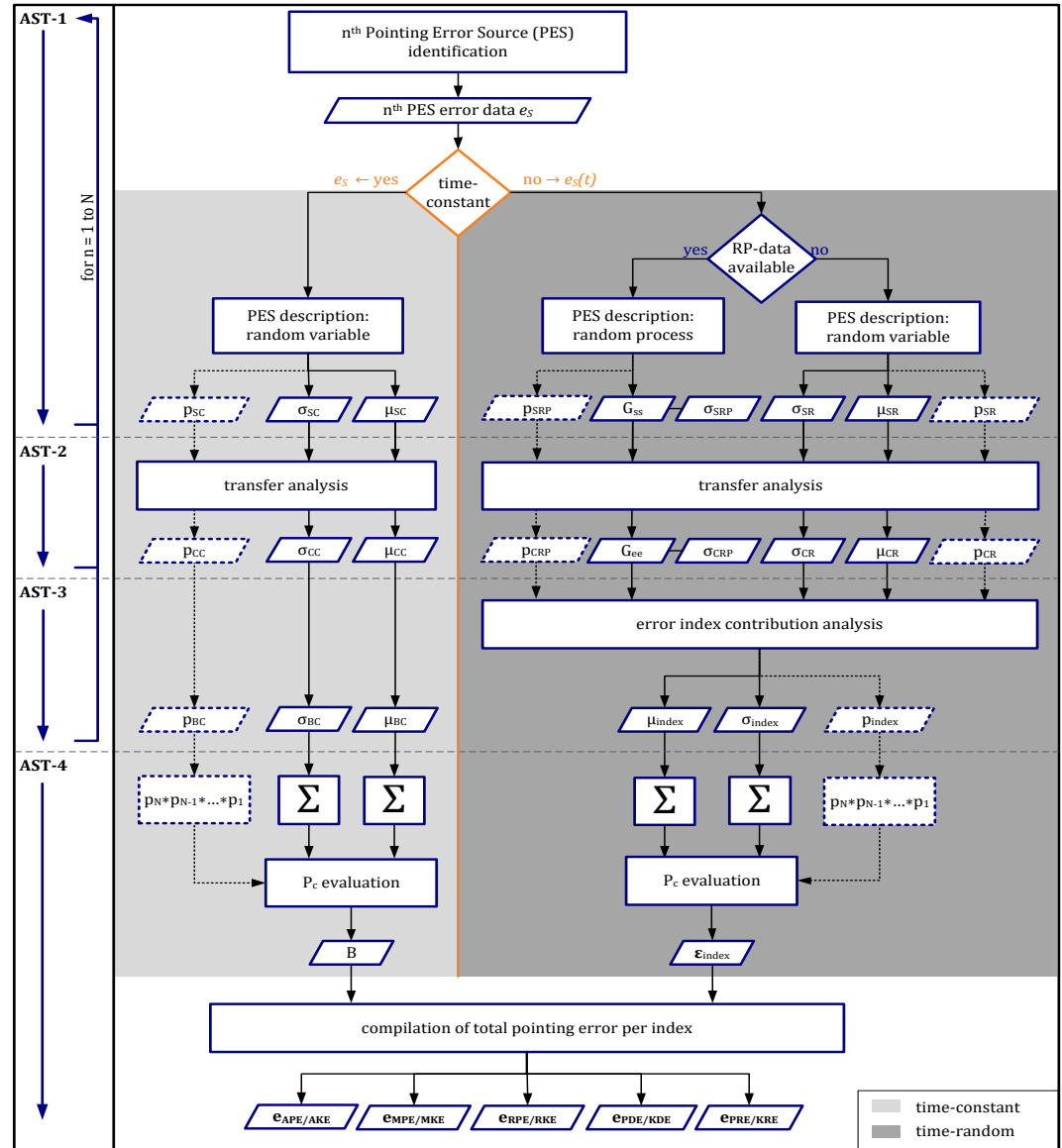
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Pointing Error Analysis and Evaluation Methodology (2/2)

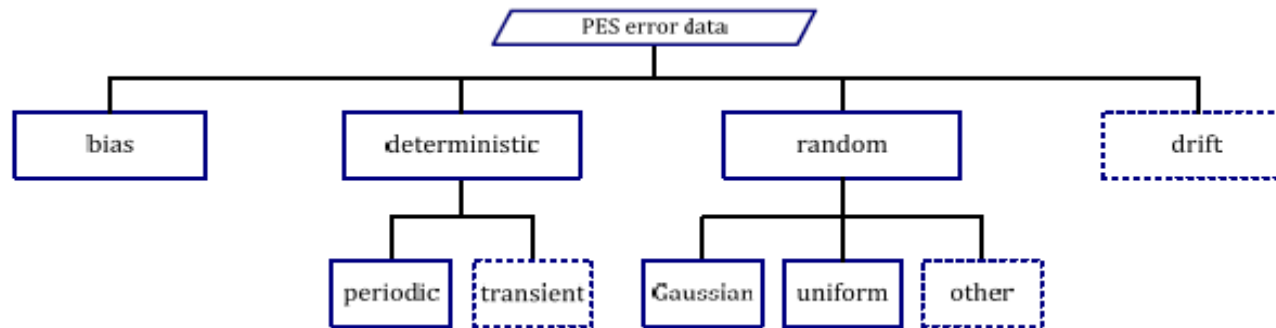
- Simplified statistical method:** analysis via variance (σ) and mean (μ) and their summation per ECSS error index under the assumption of the central limit theorem, solid lines in figure

- Advanced statistical method:** analysis by joint PDF characterisation via convolution of different error PDF $p_{...}(e)$, dashed lines in figure



AST-1: PES Characterisation (1/3)

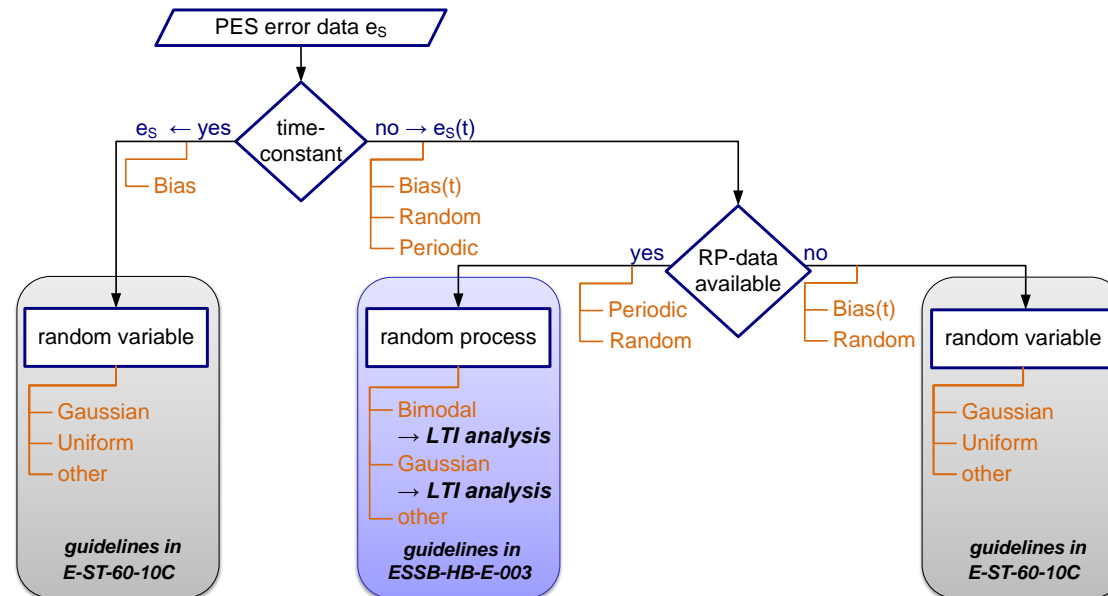
- First step for pointing error sources characterisation is their **categorisation into signal classes** (bias, periodic, random, etc.) that are analogous to those defined and considered in the ECSS standard



- PES data categorisation into signal classes allows simplifying the pointing error analysis **without loss of generality**

AST-1: PES Characterisation (2/3)

- A decision tree is defined, which provides guidelines for selecting eligible mathematical elements to describe the signal-classified PES data



- Differently from the current approach in E-ST-60-10C, description of PES as stationary random process (together with characterising its PSD) allows **exact capture** of window and stability time properties.

AST-1: PES Characterisation (3/3)

- Example: 2 PES with temporal interpretation specified

characterization steps	STR-Payload misalignment error	Gyro-Stellar Estimator (GSE) noise
temporal behavior	time-constant	time-random
signal class	bias	random
description	random variable	random process
reference document	ECSS	ESA PEE Handbook
characteristic data	$U(0, e_{max})$	$G(0, \sigma_G), G_{ee}$
temporal interpretation	$p(e) = \delta(e_{max})$	$G(0, \sigma_G)$
AST-1 output data	$\mu(e) = e_{max}, \sigma(e) = 0$	$\mu(e) = 0, \sigma(e) = \sigma_G, G_{ee}$

AST-1: PES vs Statistical Interpretation

- ESA requirement must define the considered statistical population by specifying one of the three **statistical interpretations** introduced in ECSS-E-ST-60-10C:

- Ensemble** $\text{Prob}[|e_{\max}(k)| < e_r] \geq P_c$ with $e_{\max}(k) = \max_t\{|e_k(t)|\}$
i.e. WC time but error statistically averaged over a population of S/C
- Temporal** $\text{Prob}[|e_{\max}(t)| < e_r] \geq P_c$ with $e_{\max}(t) = \max_k\{|e_k(t)|\}$
i.e. WC S/C but error statistically averaged over time
- Mixed** $\text{Prob}[|e(k,t)| < e_r] \geq P_c$
i.e. time and ensemble considered concurrently (conditional probability)

- The PEEH highlights this **important feature**: the mixed interpretation automatically yields:

- to the ensemble one if a PES does not randomly vary in time (time constant, e.g. alignment bias)
- to the temporal interpretation if it does not vary over the population (ensemble-constant, e.g. sensor typical noise).

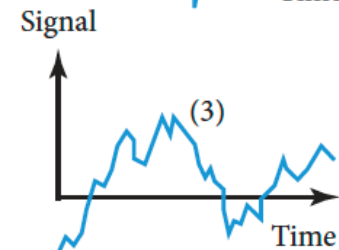
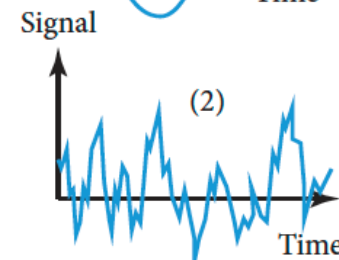
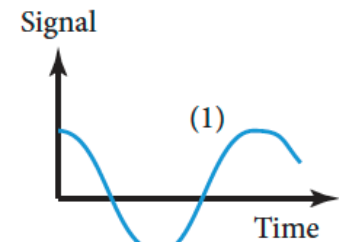
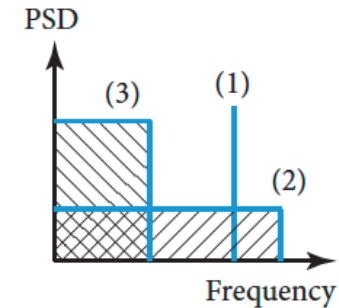
Transfer analysis: Power Spectral Density (PSD) introduction

- Power spectral density (PSD) describes the frequency content of a random process
- The mean square value of the signal is equal to the integral under the PSD curve:

$$E[x^2] = \int_0^{\infty} S_x(f) df$$

- White noise has a flat PSD curve
- Propagation through a transfer function H:

$$S_y(\omega) = |H(i\omega)|^2 S_x(\omega)$$

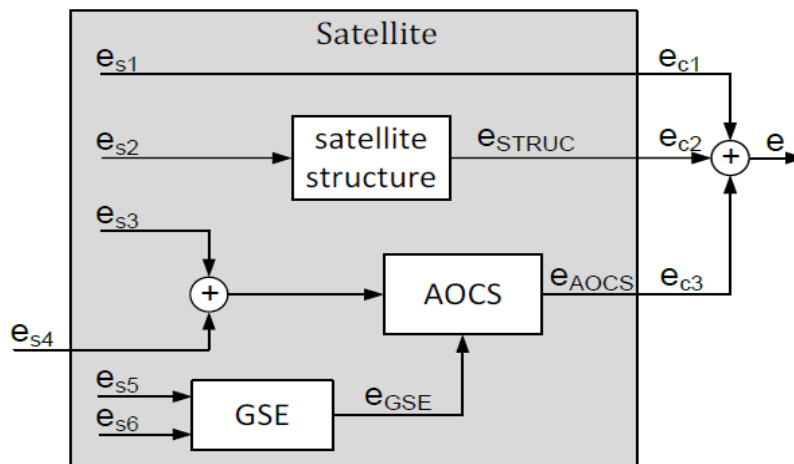


AST-2: Transfer Analysis (1/2)



Transfer from a PES point of origin to the point of interest for error calculation:

- Coordinate frame transformation
- Closed-loop control system
- Thermo-structural transformation



	PES
e_{s1}	STR-Pay. misalignment
e_{s2}	μ Vibrations
e_{s3}	thruster noise
e_{s4}	solar pressure noise
e_{s5}	STR noise
e_{s6}	gyro noise

- Transfer analysis of time-random PES described as random process can be performed in either time- or frequency-domain:
 - The time-domain approach relies on simulations that, based on the PES PDF or PSD, provide a statistical sample that can be used to characterise the PEC at the point of interest
 - The frequency-domain approach relies on linear transformation of statistical properties

- For LTI systems, the PSD transfer relation can be used:

$$G_{ee}(\omega) = |H(j\omega)|^2 G_{ss}(\omega)$$

- One **advantage of the frequency-domain approach** is that it can be **used to tune the system transfer function** H based on signal and system norms

AST-3: Error Index Contribution Analysis (1/3)

- Time-domain approach is based on simulations and described in the ECSS standard
- Frequency-domain approach provides exact contribution analysis via PSD weighting functions

$$\sigma_e^2 = \frac{1}{2\pi} \int_0^\infty G_{ee}(\omega) d\omega$$

↓

$$\sigma_{index}^2 = \frac{1}{2\pi} \int_0^\infty G_{ee}(\omega) F_{metric}(\omega) d\omega$$

Time-Windowed Error Metrics (Lucke et al. & Pittelkau)			
index:= metric		time domain	frequency domain
APE, AKE:= Absolute (ABS) Metric	$\sigma_{index}^2 :=$	$\sigma_{ABS}^2 = E[(e(t) - \mu_{ABS})^2]$	$= \frac{1}{2\pi} \int_0^\infty G_{ee}(\omega) d\omega$
MPE, MKE:= Windowed Mean (WM) Metric	$\sigma_{index}^2 :=$	$\sigma_{WM}^2(\Delta t) = E[\langle (e(t))_{\Delta t} - \mu_{ABS} \rangle^2]$	$= \frac{1}{2\pi} \int_0^\infty G_{ee}(\omega) F_{WM}(\omega, \Delta t) d\omega$
RPE, RKE:= Windowed Variance (WV) Metric	$\sigma_{index}^2 :=$	$\sigma_{WV}^2(\Delta t) = E[\langle (e(t) - \langle e(t) \rangle_{\Delta t})^2 \rangle_{\Delta t}]$	$= \frac{1}{2\pi} \int_0^\infty G_{ee}(\omega) F_{WV}(\omega, \Delta t) d\omega$
PDE, PRD, KDE, KRE:= Windowed Mean Stability (WMS) Metric	$\sigma_{index}^2 :=$	$\sigma_{WMS}^2(\Delta t, \Delta t_s) = E[\langle (e(t))_{\Delta t} - \langle e(t - \Delta t_s) \rangle_{\Delta t} \rangle^2]$ $\Delta t = \Delta t_1 = \Delta t_2$	$= \frac{1}{2\pi} \int_0^\infty G_{ee}(\omega) F_{WMS}(\omega, \Delta t, \Delta t_s) d\omega$
PDE, PRD, KDE, KRE:= Stability (STA) Metric	$\sigma_{index}^2 :=$	$\sigma_{STA}^2(\Delta t_s) = E[(e(t) - e(t - \Delta t_s))^2]$	$= \frac{1}{2\pi} \int_0^\infty G_{ee}(\omega) F_{STA}(\omega, \Delta t_s) d\omega$

AST-3: Error Index Contribution Analysis (2/3)

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Pointing Error Metric Weighting Functions F_{metric}	
Windowed Mean (WM)	
$F_{WM}(\omega, \Delta t) = \frac{2(1 - \cos(\omega\Delta t))}{(\omega\Delta t)^2}$	<p style="text-align: center;">F_{WM} with $\Delta t=0.18s$</p>
<p><i>rational approximation:</i></p> $\tilde{F}_{WM}(s, \Delta t) = \frac{2(s\Delta t + 6)}{(s\Delta t)^2 + 6(s\Delta t) + 12}$	
Windowed Variance (WV)	
$F_{WV}(\omega, \Delta t) = 1 - \frac{2(1 - \cos(\omega\Delta t))}{(\omega\Delta t)^2}$	<p style="text-align: center;">F_{WV} with $\Delta t=0.18s$</p>
<p><i>rational approximation:</i></p> $\tilde{F}_{WV}(s, \Delta t) = \frac{s\Delta t(s\Delta t + \sqrt{12})}{(s\Delta t)^2 + 6(s\Delta t) + 12}$	



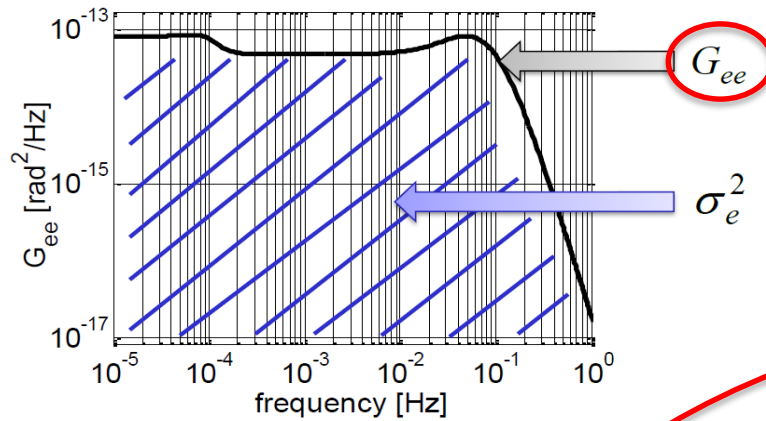
Mean error:
MPE, MKE



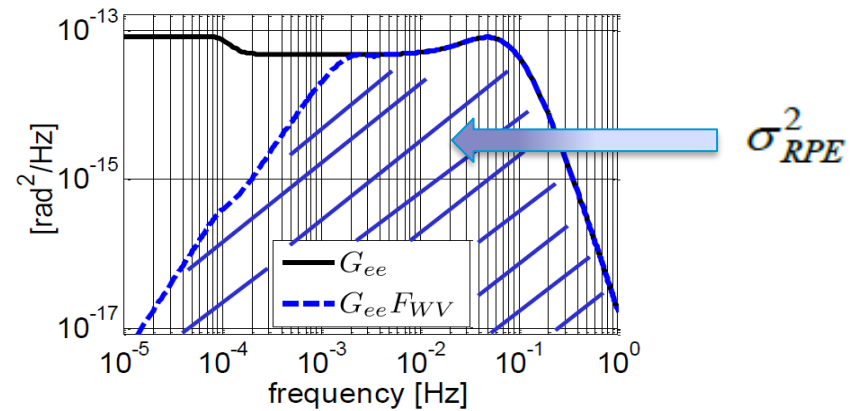
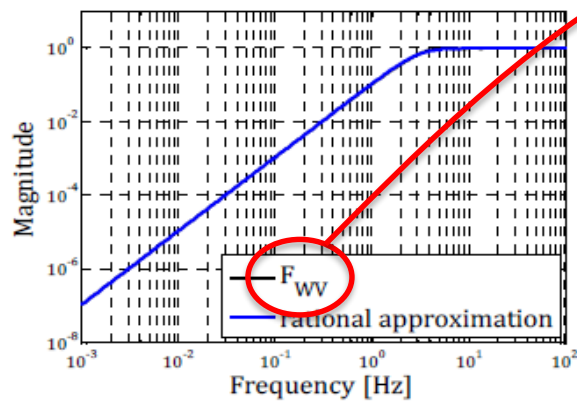
Relative error:
RPE, RKE

AST-3: Error Index Contribution Analysis (3/3)

● Example: GSE noise contribution to RKE



$$\sigma_{RPE}^2(\Delta t) = \frac{1}{2\pi} \int_0^{\infty} G_{ee} F_{WV}(\Delta t) d\omega$$



AST-4: Pointing Error Evaluation (1/3)

- The **simplified statistical method** is treated in detail in the PEEH
- In the simplified statistical method, time-constant and time-random error contributors (per index) are first summed separately

Time-Constant PEC: Summation and Level of Confidence Evaluation				
P_c evaluation		summation		
$B \leq n_p \times \sigma_B + \mu_B $		$\mu_B = \sum_{i=1}^N \mu_i$		
		$\sigma_B^2 \leq \sigma_c^2 + \sigma_{uc}^2$	<i>correlated</i>	$\sigma_c^2 \leq N \sum_{i=1}^N \sigma_i^2$ (*1) or $\sigma_c^2 \leq \left[\sum_{i=1}^N \sigma_i \right]^2$ (*2)
			<i>uncorrelated</i>	$\sigma_{uc}^2 = \sum_{i=1}^N \sigma_i^2$
B	<i>bias</i>	P_c	<i>level of confidence:</i>	
σ^2	<i>variance</i>	n_p	<i>defines max. error complying with P_c requirement of a Gaussian distribution by $n_p \times \sigma$</i>	
μ	<i>mean</i>			
N	<i>number of summands</i>			
*1: general worst case upper bound summation (see Annex for derivation). *2: upper bound summation as in ECSS standard.				

AST-4: Pointing Error Evaluation (2/3)

- Means are summed linearly
- Uncorrelated variances are RSSed
- Correlated variances summation is estimated by upper bound
- Formulas are valid under the assumption that the central limit theorem applies

TIME-RANDOM PEC: SUMMATION			
statistics	summation		
μ_{index_sum}	$= \sum_{i=1}^N \mu_i$		
$\sigma_{index_sum}^2$	group summation	signal class groups	
	$\leq \sum_{i=1}^N (\sigma_{uc}^2)_i + \sum_{i=1}^N (\sigma_c^2)_i$	correlated	$\sigma_c^2 \leq N \sum_{i=1}^N \sigma_i^2$ (*1) or $\sigma_c^2 \leq \left[\sum_{i=1}^N \sigma_i \right]^2$ (*2)
		uncorrelated	$\sigma_{uc}^2 = \sum_{i=1}^N \sigma_i^2$
σ^2	variance	N	number of summands
μ	mean		
*1: general worst case upper bound summation (see Annex for derivation).			
*2: upper bound summation as in ECSS standard [] suggested for periodic error summation.			

AST-4: Pointing Error Evaluation (3/3)

- The applicable confidence level specified in the requirements is then incorporated in the error index evaluation
- Total pointing error per index is finally obtained as sum of time-constant (only for APE, AKE, MPE, MKE) and time-random partial sums

Time-Random PEC: Level of Confidence Evaluation			
<i>index</i>	<i>P_c evaluation assuming applicability of central limit theorem</i>		<i>P_c requirement</i>
<i>APE</i> (Δt_D)/ <i>AKE</i> (Δt_D)	$\epsilon_{index} = n_p \cdot \sigma_{index, sum}(\Delta t_D) + \mu_{index} $		$\text{Prob}(e < \epsilon_{index}) \geq P_c$
<i>MPE</i> ($\Delta t, \Delta t_D$)/ <i>MKE</i> ($\Delta t, \Delta t_D$)	$\epsilon_{index} = n_p \cdot \sigma_{index, sum}(\Delta t, \Delta t_D) + \mu_{index} $		
<i>RPE</i> ($\Delta t, \Delta t_D$)/ <i>RKE</i> ($\Delta t, \Delta t_D$)	$\epsilon_{index} = n_p \cdot \sigma_{index, sum}(\Delta t, \Delta t_D)$		
<i>PDE</i> ($\Delta t, \Delta t_s, \Delta t_D$)/ <i>KDE</i> ($\Delta t, \Delta t_s, \Delta t_D$)	$\epsilon_{index} = n_p \cdot \sigma_{index, sum}(\Delta t, \Delta t_s, \Delta t_D)$		
<i>PRE</i> ($\Delta t, \Delta t_s$)/ <i>KRE</i> ($\Delta t, \Delta t_s$)	$\epsilon_{index} = n_p \cdot \sigma_{index, sum}(\Delta t, \Delta t_s)$		
<i>e</i>	<i>pointing error</i>	<i>P_c</i>	<i>level of confidence</i>
ϵ_{index}	<i>max. time-random error compliant with P_c</i>	<i>n_p</i>	<i>defines error that complies with P_c requirement of a Gaussian distribution by n_p × σ_{index}</i>
μ_{index}	<i>mean of time-random error</i>		
Δt_D	<i>drift re-set time</i>		

Pointing Error Engineering Tool (PEET)

- With the aim to complement and to ease the application in daily work of the methodology outlined in the ESA PEEH and the ECSS standard E-ST-60-10C, an accompanying SW tool called PEET has been developed
- Development timeline:
 - 2012 - Prototype of the accompanying SW tool PEET completed (see peet.estec.esa.int)
 - 2013 – Prototype update for positioning and relative pointing
 - 2017 – Fully functional operational version of the accompanying SW tool PEET completed
- Scope of PEET:
 - Translation of the engineering methodology of [ESSB-HB-E-003] into a software tool that supports system engineers
 - General error propagation and analysis tool for the assessment of any type of quantity
 - Systematic compilation of budgets for any mission including high precision pointing missions
 - Common interface for model exchange between departments and users

- The ESA PEEH together with the ECSS standard E-ST-60-10C provides all the **mathematical elements to carry out pointing error budgeting** activities
- The exact capture of window time and stability properties that is allowed by the **frequency domain methodology** described by PEEH limits the risk of over- or under-estimates that is inherent in the simplified summation rules
- The frequency domain approach also makes the PEEH methodology a useful tool to complement **the tuning activities** on system transfer functions (typically attitude estimator, attitude controller, etc.)
- The PEEH methodology can be used to assist **requirement breakdown** and allocation activities by providing sensitivity evaluation of the pointing performance with respect to the PES
- PEEH has become **an [AD] in ESA ITTs** in association with the ECSS Control Performance Standard E-ST-60-10C